Peirce and Frege on Logic

Abstract

Though Peirce and Frege played crucial roles in the development of modern logic, there has been so far little scholarly work comparing their positions on the nature of logic. I offer here a comparative study of their views on logic and I show that, although they developed their views independently from each other and with very different projects in mind, their positions are extremely similar (except for a crucial difference). I also show that the comparative study that I present is fruitful insofar as it enables us to solve a controversy concerning whether Peirce endorsed logicism or not.

1 Introduction

Although the role of Peirce in the development of logic in the 19th century has been extensively acknowledged and analyzed in contemporary literature, there has been so far little attention devoted to comparative studies of his views on logic with those of one of his most distinguished peers — namely, Frege. This absence is partially explained by the fact that Peirce’s genius led him to develop independently from other scholars all the central achievements that were established within the tradition that Frege spawned (in particular, the introduction of a revolutionary notation that allows us to integrate quantification theory and to formalize polyadic relations in logic), but it is also due to the fact that the philosophical projects that Peirce and Frege respectively undertook (and for which they developed their logical systems) are quite different. Indeed, while Frege’s interest in logic was chiefly motivated by his attempt to use it in order to ground arithmetic, Peirce’s interest in logic was motivated by his project of developing a general theory of signs.

My purpose in the present paper is to offer a comparative study of the views of Peirce and Frege on logic. The main thesis that I will argue for is that, despite the fact that both worked independently from each other and with very different projects in mind, they endorsed nevertheless very
similar views about the nature of logic—albeit with a crucial difference. In addition to showing this, I will also argue that the comparative study that I provide here allows us to cast light on the debate concerning whether Peirce was a logicist or not.

My strategy will be the following. In §2, I rehearse Peirce's conception of logic as it is presented in his early writings, particularly in the Harvard and Lowell Lectures on the Logic of Science from 1865 and 1866 (which are the first works in which he addresses in a systematic way the nature of logic)¹ and, after doing this, I turn in §3 to a characterization of Frege's view of logic. Having provided an exposition of their views, I then present in §4 a comparative assessment of them and I use it to show that their positions are very similar (except for a difference on a crucial respect). Subsequently, I argue in §5 that my comparative study enables us to cast a light on the issue of whether Peirce embraced logicism or not. Finally, in §6, I provide a brief conclusion.

2 Peirce's conception of logic

Though Peirce made many lasting contributions to different areas of philosophy, a thorough analysis of his works shows that all of these ultimately arose from his interest in logic—a passion that he developed at an early age and that he cultivated for the rest of his life. Accordingly, a successful comparative analysis of his views on logic with those of Frege requires having a good understanding of Peirce's works. Given the very extensive nature of Peirce's corpus, I will focus primarily on the conception of logic presented in his Harvard and Lowell Lectures.

How does Peirce characterize logic in these lectures? After initially rehearsing a number of traditional definitions from several Ancient and Medieval authors and rejecting them in virtue of their broadness and vagueness, Peirce focuses in the first Harvard lecture on a proposal that he attributes to Kant² to which he subsequently devotes substantial attention:

Since Kant, there has been a vast majority of the suffrages of logicians in favor of his definition which is as follows—the science

¹All the citations from Peirce used here will be drawn either from the Chronological Edition of his writings (CE) or from the Collected Papers (CP).
²Throughout this paper, I will present the conceptions of logic that Peirce and Frege endorse while making often reference to Kant. The primary reason to adopt this expository strategy is that, since Peirce and Frege were both heavily influenced by Kant, their views can be more clearly understood if we keep in mind the intellectual model that inspired them.
of the necessary laws of the Understanding and Reason—or what is the same thing—the science of the sheer Form of thought in general. (CE 1: 164)

Immediately after introducing this suggestion, Peirce remarks that it is really composed of two definitions that seem to have very different natures. As he rightly observes, in the first definition, logic is characterized in terms of the laws that govern the actual workings of certain psychological faculties whereas, in the second definition, logic is characterized in terms of a science that is concerned with a particular abstract feature—i.e., the form—that a particular class of non-mental objects—i.e., the thoughts in general—have. Peirce also contends that, even though the definitions have usually been deemed to be equivalent, there is a clear difference between them and he expresses a strong preference for the second one.

Why does Peirce prefer the second definition of logic and reject the first one? The main argument that he provides goes as follows. If one writes on the board a certain syllogism, it has a certain logical character which can be grasped by many individuals at different times. But, since the thought that I have when I consider the syllogism is different from the thought that someone else has when he considers it, there is no account, on the first view, of how the logical character of the syllogism (which is one) can be present in all the individual thoughts. In order for this logical character to be present in all the individual thoughts, Peirce argues that it cannot be just a form of any individual thought but that it also has be a form of something that is the ‘continual denominator’ of those thoughts, and he identifies this with what is written on the board. Thus, as we can appreciate, Peirce’s central reason to prefer the second abovementioned definition of logic is that the first (or psychological) definition does not allow us to explain how the laws of logic can be publicly grasped and can be used as models for correct thinking—two features which are required if logic is to be effectively used as a classificatory science (which is what Peirce intends).

Among Peirce scholars, one of the most contentious issues regarding his conception of logic is whether he considered it to be a descriptive dis-

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3Peirce (CE 1: 306) distinguishes very carefully the individual thoughts that actual human beings have from what he calls thought in general, which he describes as ‘a fiction that expresses merely the possibility of discourse’. For further discussion of the notion of thought in general in Peirce, see below.

4The importance of logic as a classificatory science that provides a framework for other scientific endeavors is emphasized in numerous places throughout Peirce’s works. For a detailed study of the importance of logic in Peirce’s project of classification of the sciences, see Kent (1987).
cipline on a par with other sciences (e.g., physics or chemistry) that aim
to characterize the world as it is or a normative discipline on a par with
ethics and aestethics. Although both interpretations of Peirce have been
considered traditionally to be mutually exclusive, a detailed analysis of his
writings reveals that he endorsed both views and that his position is per-
factly consistent. As a science, logic is for Peirce a descriptive discipline
that is concerned with uncovering and characterizing the laws that govern
certain entities (i.e., the Forms of thought in general) that, despite being ab-
stract, are no less real than the objects studied by physics. The conception
of logic as a descriptive discipline that enables us to classify the structures
different arguments and to uncover their properties is clearly reflected in
the first Lowell lecture:

And accordingly, if we wish to be able to test arguments, what
we have to do, is to take all the arguments that we can find, scru-
tiniz them and put those which are alike in a class by themselves
and the examine all these different kinds and learn their proper-
ties. Now the classificatory science of reasons so produced is the
science of Logic (CE 1: 359)

In fact, according to Peirce, it is precisely because logic is a descriptive
discipline with a specific mind-independent object of study that, once we
discover the laws that govern the forms of thought in general, these laws
(which cannot be broken) can be used as models to which our actual rea-
sonings should in principle conform:

It has been supposed that the laws of logic might be broken.
That they say ‘Thou ought’ not ‘thou shalt’, that in fact they
are statements not of fact but of debt. But what page of man’s
ledger does this ought refer to? Thought debtor to what? It is
impossible to say. But why ought we to be logical? Because we
wish our thoughts to be representations or symbols of fact. It
is evident therefore that logic applies to thought only insofar as
the latter is symbol. (CE 1: 166)

As this passage illustrates, the laws of logic clearly have a descriptive
dimension for Peirce: they state facts. But precisely because they have this

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5For instance, Michael (1979) argues that Peirce endorses clearly in his early writings
a descriptive conception of logic whereas Banks (1943), Zeman (1986) and Dilpert (2004)
maintain that he views logic as a normative discipline.
feature, they have in addition a normative import on our inferential and
classificatory practices as they express the canonical ways that all thought
processes should conform to in order to be able to reason correctly.\footnote{Peirce emphasizes the normative dimension that logic has in many passages of his
works. See, in particular, CP 1.577 where he characterizes logic as a normative discipline
and CP 1.573, CP 1.575, CP 1.611 and CP 2.82 where he defends the view that logic is a
branch of ethics.} It is
in virtue of this that the laws of logic have universal applicability: insofar
as they are non-psychological principles of general reasoning, they apply,
as Peirce (CE 1: 167) remarks, ‘not merely to what can be thought but
to whatever can be symbolized in any way. And hence [this] extends their
validity to all subjects of argumentation whatever.’

In this respect, Peirce’s view of logic is influenced by Kant’s, who also
maintains that logic has universal applicability. Moreover, the reasons that
they provide to explain the universal applicability (or generality) of logic are
almost identical: whereas Kant (A 52/B 76) maintains that logic is general
because it involves ‘the absolutely necessary rules for thought without which
there can be no employment whatsoever of the understanding’,\footnote{It
is important to keep in mind that Kant distinguishes general logic, which
abstracts completely from all content whatsoever (and is thus concerned with the forms of thought
in a way that is entirely indifferent to all their features), from transcendental logic, which
is also concerned with the forms of thoughts but is able to distinguish them on the basis
of their origin. As I will argue below, Peirce’s conception of logic seems to be influenced
by Kant’s general logic rather than by his transcendental logic.} Peirce (CE
1: 174) maintains that logic is general because it expresses ‘the conditions
which enable symbols in general to refer to objects.’ In light of this, it is
then clear that, not only Peirce and Kant accept the thesis that logic is
general, but they construe it in almost the same way: while the generality
of logic according to Kant consists in the fact that its laws are \textit{conditions
of possibility for thought as such}, the general character of logic for Peirce
consists in the fact that its laws are \textit{conditions of possibility for symbols to
have content}.\footnote{In light of this, the generality of the laws of logic for Kant and Peirce is intimately
tied to their normative character: the laws of logic are general not in the sense that it is
impossible for us to think while violating them (or to use a symbol to refer to an object
while violating them), but rather in the sense that we can only think correctly about any
subject (or use a symbol correctly to refer to an object) if we abide by them. For a more
elaborate discussion of this notion of normative generality, see MacFarlane (2002: 43-44).}

Though Kant and Peirce agree on the generality of logic, they sharply
disagree on the notion of thought they associate to logic. Whereas Kant con-
siders in the \textit{Critique of Pure Reason} (A 107) thoughts to be mind-dependent
entities that exist only insofar as they are grasped by the consciousness of a

\footnote{Peirce emphasizes the normative dimension that logic has in many passages of his
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branch of ethics.}
transcendental subject (and, consequently, views the study of logic as a part of the study of the psychological powers and limits of this transcendental subject).\footnote{This is clearly reflected in the structure of the \textit{Critique of Pure Reason} where Kant first considers in the Transcendental Doctrine of Elements the powers and limitations of the faculty of intuition (which is the object of his Transcendental Aesthetic) and he subsequently turns to examine the powers and limitations of the faculty of understanding (which is the object of his Transcendental Logic).} Peirce considers logic \textit{qua} science of the forms of thought as being divorced from psychology:

The psychological view [of logic] is that these forms are only realized in thought and that language is essential to thought. The unpsychological view is that they are forms of all symbols whether internal or external but that they only are by virtue of possible thoughts. In short, I say that the logical form is already realized in the symbol itself; the psychologist says that it is only realized when the symbol is understood. (CE 1: 165-166)

This passage shows that logic is formal for Peirce insofar as its laws are concerned with certain relations—namely, the forms of thoughts. But it also shows that Pierce endorses, \textit{contra} Kant, the view that these forms are objective, mind-independent entities: they are already present in the symbols without requiring a subject that grasps them. Thus, though Peirce follows Kant in accepting the view that logic is formal since its laws are concerned with certain relations between thoughts in general, it is clear that he distances himself from Kant insofar as he rejects that logic is a part of transcendental psychology.\footnote{It is important to observe here that, though Kant accepts that logic is part of transcendental psychology, he rejects (1992: 252) in the Prolegomena to the Vienna logic the view that empirical psychology is involved in logic because ‘since [psychology] is an empirical science, there would arise from this a science of how we think under various hindrances, not of how we ought to think. There would be nothing but contingent and natural laws. But that is not what we are asking about. Logical rules must be derived from the necessary use of the understanding.’}

Now, in addition to accepting the view that the laws of logic are formal insofar as they are concerned with the forms of thoughts, Peirce is also indebted to Kant in another respect. One of the central characteristics of logic according to Kant (A 54/B 78) consists in the fact that ‘it abstracts from all content of the knowledge of understanding and from all the difference in its objects, and deals with nothing but the pure form of thought.’ Keeping in mind that Peirce agrees that logic is the science of the sheer Form of thought in general, it is not difficult to appreciate that he endorses the view

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that logic abstracts from all content when we consider the way in which he understands the notion of form within the framework of logic:

A form is the relation between the representation and thing pre-scinded from both representation and thing. (CE 1: 274, my emphasis)

Moreover, if we also bear in mind that Peirce’s characterization logic as the science that is concerned with the Form of thought in general, a further piece of evidence that shows that Peirce viewed logic as a discipline that abstracts from all content comes to light when we consider his view of the notion of thought in general:

Thought in general too is a very different thing from a thought in the mind. In the first place, it does not exist in the individual mind but it is common to you and me. In the second place it is prescinded from intuition and so reduced to an ens rationis which is one of Kant’s four species of nothing. (CE 1: 311)

This passage is of crucial importance because it not only provides further evidence for the view that the laws of logic are mind-independent for Peirce, but it also strongly supports, via the reference to Kant, the thesis that the laws of logic are indeed devoid of all content. Indeed, since Kant characterizes an ens rationis in the Critique of Pure Reason (A 290/B 347) as ‘a concept without an object’, it is then clear that, if the laws of logic are exclusively concerned for Peirce with relations between entities that are empty shells and to which no genuine object corresponds, they have to abstract from all content.

Finally, if we consider carefully Peirce’s claim (which I quoted above) that the laws of logic express ‘the conditions which enable objects in general to refer to objects’, it is clear that they are devoid of content because they are general principles that any symbol must conform to in order to refer (i.e., in order to have content). Thus, a law of logic such as the Law of Non-contradiction has no content for Peirce because it does not correspond to any object: it is merely a general condition that any symbol has to fulfill to refer to some object.\footnote{It is in virtue of this fact that Liar sentences cannot have any content for Peirce (at least in his early writings): since they violate the Law of Non-contradiction, they cannot refer to anything. For a more detailed discussion of Peirce’s views on the Liar Paradox, see Atkins (2011).} In other terms, the laws of logic are devoid of...
content for Peirce because they are meta-semantic principles that regulate (and are prior to) the attribution of any content whatsoever.\textsuperscript{12}

This feature (i.e., abstraction from all content) plays a very important role for Peirce to the extent that he relies precisely on it to explain why logic is \textit{a priori}. Indeed, since the laws of logic are formal in the sense that they abstract from all content, Peirce (CE 1: 422) maintains in the fourth Lowell Lecture that, because of this, they ‘do not depend on any particular state of things; and hence we say we have not derived them from experience’. In light of this, we can appreciate that the abstraction from all content that the laws of logic exhibit is extremely important from an epistemic viewpoint since this is what enables Peirce to explain their independence from experience.

Let me sum up the main points made in this section. For Peirce, logic is a science that is formal because it is concerned with the forms of thought in general and its laws are general insofar as they are applicable to everything that is symbolizable (and not merely to everything that is thinkable). To be more specific, logic is formal because its laws reflect independently of the workings of the mind of any subject whatsoever certain forms, which are relations that are characterized by completely abstracting from all content insofar as they prescind from their relata. And it is precisely in virtue of the fact that they abstract from all content that they are known \textit{a priori}. Having specified the nature of logic for Peirce, let us consider Frege’s conception.

3 Frege’s conception of logic

After recounting in the previous section Peirce’s conception of logic, it is now time to turn to Frege. Frege’s most extended discussions of the nature of logic and of its laws occur in the \textit{Foundations of Arithmetic (FA)} and in the \textit{Basic Laws of Arithmetic (BLA)}. In particular, the introduction of \textit{BLA} contains a very detailed exposition of Frege’s view about logic. In order to appreciate the core elements of it, allow me to quote a long but extremely revealing section:

\begin{quote}
It will be granted by all at the outset that the laws of logic ought to be guiding principles of thought in the attainment of truth;
\end{quote}

\textsuperscript{12}Peirce endorses in other passages of his early writings the thesis that logic is devoid of content. For instance, he maintains in ‘Upon the Logic of Mathematics’ (CE 2: 67) that ‘the principles [of logic] considered as speculative truths are absolutely empty and indistinguishable’ and, in this paper, he references an earlier paper, ‘On the Natural Classification of Arguments’, where he (CE 2: 25) argues that ‘every logical proposition considered as a proposition will be found to be quite empty.’
yet this is only too easily forgotten and here what is fatal is the double meaning of the word ‘law’. In one sense a law asserts what it is, in the other it prescribes what ought to be. Only in the latter sense can the laws of logic be called ‘laws of thought’: so far as they stipulate the way in which one ought to think. Any law asserting what is can be conceived as prescribing that one ought to think in conformity with it, and is thus in that sense a law of thought. This holds for laws of geometry and physics no less than for laws of logic. The latter have a special title to the name ‘laws of thought’ only if we mean to assert that they are the most general laws, which prescribe the way in which one one ought to think, if one is to think at all. But the expression ‘law of thought’ seduces us into supposing that these laws govern thinking in the same way that the natural laws govern events in the external world. In that case they would be laws of psychology: for thinking is a mental process. [...] I understand by ‘laws of logic’ not psychological laws of takings-to-be-true, but laws of truth. [...] They are boundary stones set in an eternal foundation, which our thought can overflow but never displace. It is because of this that they have authority for our thought if it would attain to truth. (1967: 12-13)

This passage reveals in great detail Frege’s view. After initially characterizing the laws of logic as ‘laws of thought’ that guide us towards truth, Frege observes that his position may generate some confusion in virtue of an ambiguity in the word ‘law’, which may be understood in two different ways. According to the first one, the laws of logic are ‘laws of thought’ in the sense that they describe our actual ways of reasoning and the mental processes and events involved in the exercise of our psychological faculties. Frege rejects this view for reasons similar to the ones that Peirce voices. If this view were correct, then the laws of logic would be just like the laws of fashion to the extent they would be (at best) the result of some contingent feature about the world (such as our mental constitution) and they would accordingly lack genuine normative force.¹³

In contrast, according to the other view (which Frege subscribes to), the laws of logic are ‘laws of thought’ in the sense that they prescribe how we ought to reason so that the thoughts we entertain are directed towards truth.

¹³The normativity of the laws of logic vis-à-vis our actual inferential practices is not a primitive or basic feature for Frege, but rather a characteristic that derives from a more fundamental feature — namely, the fact that they are, in Frege’s words, ‘laws of truth’.
Now, since logic is, according to Frege, a discipline the laws of which specify how we must think to attain truth, the following question arises: which feature (if any) enables the laws of logic for Frege to have this normative character? As the end of the abovementioned quote of the introduction of BLA suggests, truth is what provides the laws of logic with their normative force to the extent that it is, as Frege observes, an ‘eternal foundation’. To be more precise, it is because truth is, according to Frege, an objective constituent of the world that is the object of study of the laws of logic that these very laws have a normative import on the actual reasonings and inferences that we perform:

All sciences have truth as their goal, but logic is concerned with it in quite a different way: logic has much the same relation to truth as physics does have to weight or heat. To discover truth is the task of sciences; it falls to logic to discern the laws of truth. The word ‘law’ is used in two senses. When we speak of moral or civil laws we mean prescriptions which ought to be obeyed, but with which actual occurrences are not always in conformity. Laws of nature are general features of what happens in nature, and occurrences in nature are always in accordance to them. It is rather in this sense that I speak of laws of truth. […] From the laws of truth there follow prescriptions about asserting, thinking, judging, inferring. (1984: 351)

Thus, as we can see, Frege endorses a non-psychological conception of logic (i.e., a conception according to which its laws are objective entities, completely independent from any mechanisms or processes of thought), and he also holds that the laws of logic have a dual nature —i.e., they are both descriptive and normative. To be specific, they are descriptive in the sense that they are concerned with a certain mind-independent entity (i.e., truth) and they are normative in the sense that, since they aim to

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14Burge (2005: 124) has stressed that, according to Frege, truth not only is an ultimate ground upon which the laws of logic are based but also an all-pervasive component that ‘must be in the ontology of all the parts of logic’. I agree with Burge’s assessment on this issue and, as I will argue below, it is precisely because truth is omnipresent in all the parts of logic (in particular, in inferences, assertions and judgments) and because all these parts of logic are constituents of all the sciences that logic is a general discipline.

15Frege’s rejection of the psychological view of logic is neatly expressed by the first principle he embraces in the introduction of FA (1968: x): ‘Always separate sharply the psychological from the logical, the subjective from the objective.’

16This dual nature of the laws of logic according to Frege is emphasized in several places in recent literature. See, e.g., MacFarlane 2002 and Burge 2005.
study truth, they can be used to evaluate all the assertions, judgments and inferences in which the notion of truth is deployed.

In addition to rejecting the psychological conception of logic that Kant adopts, there is another respect in which Frege’s view of logic differs from Kant’s. While Kant holds, as I have pointed out in the former section, that the laws of logic abstract entirely from all content, Frege explicitly rejects this thesis because, if logic were to prescind from all content whatsoever, this would make it completely indifferent to any concepts:

Just as the concept point belongs to geometry, so logic, too, has its own concepts and relations; and it is only in virtue of this that it can have a content. [...] To logic, for example, there belong the following: negation, identity, subsumption, subordination of concepts. And here logic brooks no replacement. It is true that in an inference we can replace Charlemagne by Sahara, and the concept king by the concept desert, in so far as this does not alter the truth of the premises. But one may not thus replace the relation of identity by the lying of a point in a plane. Because for identity there hold certain logical laws which as such need not be numbered among the premises, and to these nothing would correspond on the other side. Consequently, a lacuna might arise at that place in the proof. (1984: 338)

As we can see, Frege maintains here that logic must have a certain content in virtue of the fact that there are certain concepts which are essential to it —in particular, the concepts of negation, identity, subsumption and concept subordination. These concepts are, according to Frege, that which enables logic to have a content to the extent that, if we substitute these concepts by other concepts, logic can no longer be used as a discipline that enables us to offer an account of why certain truths are justified by and derive from other truths. Indeed, as Frege argues, if we were to substitute the concept of identity by the concept of a point (or any other concept), we would no longer be able to explain why one can validly infer from the judgments that $a$ is identical to $b$ and that $a$ is $F$ the judgment that $b$ is $F$: we would have, as he puts it, a ‘lacuna in the proof’ since the concept of identity is governed by certain laws that are specific to it and that explain why the abovementioned inference is valid. In virtue of this, it is clear for Frege that logic cannot abstract from all content whatsoever because, if that were case, one could in principle replace within logic any concept by any other concept —and that is clearly impossible without making logic unable to tell
apart truth-preserving from non truth-preserving inferences. Consequently, given that logic cannot be completely indifferent to all concepts whatsoever under pain of no longer being able to account for the justificatory ordering of truths in valid inferences, it cannot be devoid of all content.\footnote{Though Frege (1968: 83) maintains in §70 of \textit{FA} that ‘what is of concern to logic is not the special content of any relation but only its logical form’, he does not embrace Kant’s view that logic abstracts from all content to the extent that he distinguishes the special content from the general content of a logical law. Thus, although Frege would concede that a law of logic such as $(\forall x)(x=x)$ lacks special content insofar the variables in it may adopt any value, he would maintain that it has a general content because the identity relation involved in it cannot be arbitrarily replaced by any other relation without potentially invalidating the truth-preserving inferences in which this law of logic is employed.}

Since the laws of logic must have content according to Frege, two questions arise: what is their content? And why do they have the particular content that they have? An answer to the first question can be found in the aforementioned opening lines of ‘The Thought’: if the laws of logic bear to truth the same relation that the laws of physics (and, in particular, the laws of thermodynamics) bear to heat, then it is clear that the content of logic is truth. But this still leaves the second question unanswered. In order to address it, it is crucial to keep in mind that Frege’s logicist project involved, as he (1968: 2) himself acknowledges in section §2 of \textit{FA}, the development of a justificatory ordering of truths which allows, after identifying certain primitive laws which cannot be demonstrated, an ‘insight into the dependence of truths on one another’. Thus, since the laws of logic are taken by Frege to be either primitive principles that either admit no proof or principles that are derivable from other basic principles (and definitions) exclusively through truth-preserving inferences, it is clear that their content must be in accordance with their primitive, undefinable character. And that is indeed the case insofar as Frege (1984: 353) maintains in ‘The Thought’ that “the content of the word ‘true’ is \textit{sui generis} and indefinable.”

Considering the fact that the laws of logic have content for Frege, one may then wonder if he disagrees with Kant’s claim that they have universal applicability (or generality). Indeed, it would seem that, if the laws of logic have a specific subject matter, they cannot be universally applicable since they must be restricted to that specific subject matter. Now, a thorough reading of Frege’s works shows that he does maintain that the laws of logic are indeed general, and that this does not conflict necessarily with the fact that they are concerned with a specific object.

In order to appreciate this clearly, it is important to keep in mind that Frege characterizes truth in the opening paragraph of ‘The Thought’ quoted
above as both the \textit{objective} of every science and as the \textit{object} of study of the laws of logic. If truth is indeed the objective of every science, it clearly must have a general character in the sense that it can be used, as Frege (1972: 104) remarks, in connection with a certain specific language (i.e., Frege’s concept-script) that is expressly designed to represent the formal relations holding between different propositions that make up scientific theories to ‘test in the most reliable manner the validity of a chain of reasoning and expose each presupposition that tends to creep in unnoticed.’

The generality of truth can be more clearly seen when we consider the following facts: since the notion of truth is represented within Frege’s concept-script as a combination of two different elements (namely, the concept stroke and the judgment stroke) and since the resulting symbol ‘⊢’ must be attached according to Frege (1972: 113) to all judgments whatsoever, it is clear that truth must have general character insofar as it is present in any judgment that is a part of any science. In virtue of this, if we keep in mind that Frege also characterizes truth in the opening paragraph of ‘The Thought’ as the object of study of the laws of logic, one can appreciate that the laws of logic for Frege are general in the sense that, since their object of study is present in every science because it is a constituent part of every judgment (and of every inference involving judgments), they can be used to evaluate all the specific instances of judgment and inference that take place within any domain of inquiry.

Having previously shown that Frege holds that logic does not completely abstract from content, one may be tempted to ask if the laws of logic are known \textit{a priori} for him given that the \textit{a priori} character of logic is traditionally explained in virtue of the fact that logic does not make any distinction among its objects because it abstracts completely from all content.\footnote{This thesis is clearly maintained by Kant (1992: 528) who affirms that ‘we can have insight into these rules [of logic] \textit{a priori}, i.e., independent of all experience, because they contain merely the conditions for the use of the understanding in general, without distinction among its objects, be that use pure or empirical.’ And, as we have seen at the end of the previous section, it also appears to be endorsed by Peirce.} An analysis of his works shows that he does accept maintain that the laws of logic are known \textit{a priori}, but his conception of \textit{a priority} is slightly different from the one that Kant endorses. Indeed, though the \textit{a priority} of the laws of logic consists for Kant and Frege in the fact that they have a privileged epistemic status (i.e. their justification is independent from experience), their respective accounts of this status are quite different.

Whereas Kant maintains that the \textit{a priori} character of logic arises from the fact that its laws abstract from all content, Frege holds that the laws of
logic are *a priori* because they are either exclusively derivable via deductive inferences from general laws that neither admit nor need any proof or they are themselves general laws that are self-evident.\(^{19}\) In light of this, the *a priori* of the laws of logic does not stem for Frege, as it does in the case of Kant, from a semantic characteristic (i.e., abstraction from all content) but rather from an epistemic one (i.e., either being derivable from self-evident and indemonstrable general laws that prescribe how one ought to think or being a self-evident and indemonstrable general law).\(^{20}\)

Before turning to a comparative analysis the views of Peirce and Frege on the nature of logic, let me summarize briefly Frege’s position. Frege’s conception of logic is greatly influenced by Kant’s system in certain respects—in particular, Frege agrees with Kant’s thesis that the laws of logic are ‘laws of thought’. However, he rejects the view that the laws of logic reflect the actual mechanisms or processes through which we reason. The laws of logic are not laws of psychology but laws of truth that have a general content. Moreover, they have a dual nature for Frege: they are descriptive because they characterize a certain objective, mind-independent entity (i.e., truth) and it is in virtue of this that they can be used as normative principles to evaluate different reasonings. In fact, it is because truth is present in every judgment and every inference that is made within any domain of inquiry that the laws of logic (which are laws of truth) have a general character: the ubiquity of truth is precisely what allows us, according to Frege, to use the laws of logic as normative standards for all the judgments and inferences we make. Finally, though the laws of logic are *a priori* for Frege, his account of their *a priori* character is different from the one Kant provides: whereas the *a priori* of the laws of logic stems for Kant from the fact that they are indifferent to all objects since they abstract from all content whatsoever, their *a priori* for Frege stems from the fact that bear certain relations to self-evident general laws within a canonical justificatory ordering of truths.\(^{19}\)\(^{20}\)

\(^{19}\)Frege (1968: 4) characterizes in §3 of *FA* the notion of *a priority* in the following way: ‘For a truth to be *a posteriori*, it must be impossible to construct a proof of it without including an appeal to facts, i.e., to truths which cannot be proved and are not general, since they contain assertions about particular objects. But if, on the contrary, its proof can be derived exclusively from general laws, which themselves neither need nor admit proof, then its truth is *a priori*.’

\(^{20}\)The notion of derivability used by Frege in the definition of *a priority* is not that of actual derivability, but rather that of *ideal* or *canonical* derivability that exists, independently of any mental activity, within an objective justificatory ordering of truths. For discussion of this issue, see Burge (2005: 361).
4 Comparing Peirce and Frege

In the previous two sections, I have provided a brief exposition of what I take to be the main features of logic as both Peirce and Frege conceive it. As I have shown, both owe a great intellectual debt to Kant. For instance, both take the laws of logic to have a normative import on our thought. However, it is important to observe that both have a conception of logic and of the relation that it bears to thought different from Kant’s.

Indeed, whereas Kant (1992: 529) conceives logic essentially as a canon (i.e., a body of rules) that is a sense prior to and independent from any science insofar as it just provides a series of norms that one is is required to follow in order to reason correctly about any subject whatsoever, both Peirce and Frege conceive logic as a specific science (i.e., a body of truths) which is primarily concerned with describing the features of certain portions of reality. In virtue of this, though Peirce and Frege agree with Kant’s view that logic is a normative discipline, both reject the view that the normativity of logic is primitive. For Frege and Peirce, the normative character of logic stems precisely from the fact that logic is a descriptive inquiry concerned with an objective, mind-independent subject-matter: it is because the laws of logic reflect certain facts about the nature of the world that we want our actual reasonings and inferential practices to conform to them.

Now, in addition to the fact that Peirce and Frege agree that logic is a science (a not a mere canon) that has both a descriptive dimension and a normative one, their views of logic are similar in another respect (which is at odds with Kant’s conception): both reject the psychological characterization of logic. This common attitude with respect to this characterization is due to the same reason in both cases: Peirce and Frege alike adopt what Proust (1989: 156) has called an ‘onto-transcendental’ stance with respect to the notion of thought.

What does this ‘onto-transcendental’ stance consists in? In a nutshell, thoughts are no longer taken to be psychological entities that exist in virtue of being understandable by a transcendental subject, but they are considered to be independent, self-standing objects that can be grasped and shared by different subjects. This stance is essential for both Peirce and Frege in order to be able to maintain that logic is a discipline that is both descriptive and normative. Indeed, if thoughts are not considered as objective, mind-independent entities, then logic (considered as a discipline that is exclusively concerned with the ‘laws of thought’) can, as Kant (1992: 529) observes, be used ‘merely for passing judgment and for correcting our cognition, but not for expanding it.’
Though Peirce and Frege depart from their Kantian roots insofar as they equally reject the psychological conception of logic, both accept, following Kant, the thesis that logic is a general discipline. Moreover, they also construe, as Kant does, the generality of logic in a normative sense because that the laws of logic are for both constitutive of the widest domain of all—a fact which makes them applicable as normative standards to everything that falls within that domain.\footnote{Although Peirce and Frege agree on the normative nature of the generality of logic, they disagree on what is the widest possible domain. For Peirce, it is the domain of what is symbolizable but, for Frege, it is the domain of what truth is applicable to.} Indeed, for Frege, the laws of logic are general as they can be applied in every field of enquiry to assess any judgment or inference whatsoever precisely because truth (which is the object of the laws of logic) is present in every judgment and every inference. In virtue of this, the generality of logic consists for Frege in the fact that its laws can be used to assess everything that truth applies to. Similarly, in the case of Peirce, the generality of logic is due to the fact that, as its laws govern everything that is symbolizable, these laws can be applied as normative standards for all subjects of argumentation in which symbols can be used. And, considering that everything is symbolizable for Peirce, the laws of logic are then general.

Let me pause to summarize the results of my comparative study. As I have shown above, Peirce and Frege have very similar views concerning the nature of logic since both accept the following theses:

\begin{align*}
(T_1) & \text{ The laws of logic constitute a science and not a mere canon.} \\
(T_2) & \text{ The laws of logic are not psychological laws: logic is not a part of psychology.} \\
(T_3) & \text{ The laws of logic are both descriptive and normative.} \\
(T_4) & \text{ The normativity of the laws of logic is not primitive: it stems from their descriptive character.} \\
(T_5) & \text{ The laws of logic are general: they have the widest possible domain of application.} \\
(T_6) & \text{ The generality of the laws of logic is normative: the laws are general in the sense that they can be used to assess everything that falls under the domain of logic.}
\end{align*}

Considering that Peirce and Frege worked in isolation of each other, the extent of their agreement on the nature of logic is quite remarkable. The
convergence of their views, however, is not absolute. In particular, there is a very important issue on which Peirce and Frege disagree with each other:

\[(T_7) \text{ The laws of logic abstract from all content.}\]

As we previously saw, whereas Peirce’s conception of logic as the science of the Forms of thought in general precludes logic having any content whatsoever since the relations that logic studies prescind completely from their relata, Frege is committed to the thesis that logic must have some content in virtue of the fact that there are certain concepts and relations that are specific to it. Although this issue may seem to be a minor difference, I think that it has a crucial importance insofar as it can help us to accomplish two goals: (i) showing that Peirce’s epistemology of logic is much more Kantian than Frege’s and (ii) casting light on the debate regarding whether Peirce endorsed logicism or not.

Indeed, given that the motivations of Peirce and Frege to endorse the a priority of logic are different as I have shown above (for Peirce, the a priority stems from the fact that logic abstracts from all content whereas, for Frege, it is a consequence of its laws being exclusively derivable from general laws plus definitions), one can argue that Peirce’s conception of the a priority of logic is more Kantian than Frege’s because Kant views the a priority of logic as stemming from its absence of content whereas Frege’s conception of a priority is closer to Leibniz’s insofar as both endorse the thesis that a priority of the laws of logic stems from their bearing certain relations to self-evident truths.\(^{22}\) Finally, in order to appreciate how the disagreement concerning \((T_7)\) can help us to solve the issue of whether Peirce embraced logicism or not, let us move to the next section.

5 Was Peirce a logicist?

I have presented a study of Peirce and Frege that shows that, even if their views about logic coincide in many respects, they differ on one crucial issue: whereas Peirce maintains that logic abstracts from all content of logic, Frege rejects this view. In this section, I want to establish that this difference allows us to shed light on a controversy regarding Peirce’s position vis-à-vis logicism.

\(^{22}\)For a persuasive case in favor of Frege’s intellectual debt vis-à-vis Leibniz regarding the a priority of logic, see Burge (2005).
Haack (1993) has argued that, even if Peirce rejects logicism, he seems to harbor some logicist sympathies insofar as there are some passages in his works in which he seems to endorse the thesis that mathematics can be reduced to certain general propositions of logic. In order to account for the tension between these passages and those in which he rejects logicism, Haack has claimed that logicism is best conceived as consisting of two central theses: (L\textsubscript{1}) arithmetic/mathematics is reducible to logic (which is a formal thesis) and (L\textsubscript{2}) the epistemological foundations of arithmetic/mathematics lie in logic (which is an epistemic thesis). After establishing this distinction, Haack remarks that, since the textual evidence seems to support the claim that Peirce rejects (L\textsubscript{2}) but accepts (L\textsubscript{1}), there seems to be a tension on Peirce’s view about logicism. In order to solve this tension, she explores several alternatives and ultimately suggests the following solution: there is an ambiguity in the notion of logic that Peirce uses in his works. When Peirce uses the term ‘logic’, he refers sometimes to a formal deductive system (which Haack refer to using the label ‘logic’) and sometimes to a general theory of reasoning (which she denotes using the label ‘LOGIC’). According to Haack, once we draw this distinction between ‘logic’ and ‘LOGIC’, we can construe Peirce as holding the consistent view that arithmetic/mathematics is reducible to formal deductive logic and that the epistemological foundations of arithmetic/mathematics do not lie in the general theory of reasoning. In virtue of this, Haack concludes that Peirce was indeed a logicist—although of a different stripe than Frege who holds that (L\textsubscript{1}) and (L\textsubscript{2}) stand or fall together.

Though Haack’s proposal is an attractive suggestion to explain away the tension between the different observations made by Peirce, it has met a number of objections. In particular, Houser (1993) has contended that, according to Haack, (L\textsubscript{1}) involves two different clauses —namely, the clause that all special concepts of arithmetic/mathematics are definable in purely logical terms and the clause that all special theorems of arithmetic/mathematics are derivable from purely logical principles—and that neither clause is accepted by Peirce. I will not rehearse Houser’s arguments here; instead, I will simply grant his claim that Peirce does not accept (L\textsubscript{1}). In spite of this,

\footnote{In particular, see his remarks concerning Dedekind in CP 4.239 as well as the following passages: CP 2.10, CP 2.181, CP 2.197, CP 3.427, CP 4.228, CP 4.228, CP 4.242 and CP 4.243.}

\footnote{In particular, see CE 1: 386, CE 2: 59-60, CE 4: 299, CP 4.88 and CP 4.93.}

\footnote{There are two versions of (L\textsubscript{1}) and (L\textsubscript{2}) which correspond, according to Haack, to two different versions of logicism: a weak one that only concerns arithmetic and a strong one that concerns the whole of mathematics.}
I think that this does not undermine Haack’s general conclusion —namely, that Peirce endorsed a certain version of logicism.

Here is why. According to what I have shown, Peirce accepts (T7), so it would be very awkward for him to maintain that arithmetic/mathematics is reducible to something that lacks content. But, if Peirce does not endorse either (L1) or (L2), how can he be a logicist? At this stage, it is important to bear in mind that both Haack and Houser assume that being a logicist requires embracing either both (L1) and (L2) or at least one of the theses. However, some authors have suggested that this characterization of logicism is misleading. Though logicism has been traditionally characterized in terms of (L1) and/or (L2) because its most distinguished exponents (i.e., Frege and Russell) endorsed versions of logicism that involve either both theses or at least one of them, there is a third thesis (which is a semantic thesis) that is characteristic of logicism:

\[(L_3)\] Arithmetical/mathematical truth is logical truth.

(L3) is often overlooked in virtue of the fact that it is considered to follow in a straightforward way from (L1); indeed, if the theorems of arithmetic/mathematics can be deduced from purely logical principles, it seems that mathematical truths are just an extension of logical truths. However, (L3) is logically independent of (L1), as the fact that some authors (e.g., Wittgenstein 1922) endorse (L3) but reject (L1) suggests. Indeed, even though Wittgenstein clearly disagrees in the Tractatus with the attempts of both Frege and Russell to derive either arithmetic or mathematics from logic, he nevertheless seems to defend the weak version of (L3) since he presents a unified account of logical truths and arithmetical truths in terms of certain kinds of operations —operations on propositions in the case of logical truths and operations on numbers in the case of arithmetical truths.

The gist of Wittgenstein’s proposal is this: after he introduces in 5.2522 the sign \('[a, x, O'x]' as a general term to refer to a series of forms in which

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26For instance, Landini (2007: 147) maintains that ‘logicism is the thesis that mathematics is reducible to logic, or so it is often said. Unfortunately, this characterization is misleading. [...] as we shall see, “reduction” in the sense of a derivation from axioms, is not essential to logicism.’

27Wittgenstein’s strongest objections to the logicist programs of Frege and Russell can be found in 4.1272, where he rejects Frege’s conception of numbers as objects and in 6.1232, where he criticizes Russell’s claim that the Axiom of Reducibility is a part of logic.

28For a more detailed account of Wittgenstein’s unified treatment of arithmetical and logical truths in the Tractatus, see Pasquale 1998.
‘the first term of the bracketed expression is the beginning of the series of forms, the second is the form of a term \( x \) arbitrarily selected from the series, and the third is the form of the terms that immediately follows \( x \) in the series’, he first argues in 6 that this purely formal device can be used to characterize the general form of a proposition, which is \([\bar{\eta}, \bar{\xi}, N(\bar{\eta})]\), and he subsequently argues in 6.03 that it can also be used to characterize the general form of an integer, which is \([0, \xi, \xi+1]\). Since the characterizations of both propositions and integers involve the abovementioned formal device (which, according to Wittgenstein, has the basic structural features of an operation), both the truth of arithmetical statements and logical laws (which are respectively composed of propositions and integers) can then be accounted in terms of certain types of operations performed on primitive elements.

It is my contention that, though Peirce is neither an \( L_1 \)-logicist nor an \( L_2 \)-logicist, he embraces a version of \( L_3 \)-logicism similar in certain respects to Wittgenstein’s view. Indeed, when we consider carefully how Peirce views both mathematical and logical truths, it is clear that, just as Wittgenstein, he does not aim to reduce the former to the latter, but rather to give a unified account of them. Peirce identifies the basis of this unified account clearly in the following passage:

Moreover, according to the opinion defended in the present treatise, logical truth is grounded upon a sort of observation of the same kind as that upon which mathematics is grounded. (CP 2.82)

For Peirce, both mathematical and logical truths are grounded on a sort of observation, which is not empirical observation, but rather observation of a certain kind of abstract entities (i.e., Peirce’s icons), which exist only as ‘image[s] in the mind’ (CP 4.447). Moreover, according to Peirce, the observation and manipulation of icons constitutes not only the basis upon which logical and mathematical truths are grounded, but also the basis of all inferences either in mathematics or in logic:

In the case of a rational inference, we see, in an icon which represents the dependence of the icon of the conclusion upon the icon of the premise, about what that class of inference is. (CP 2.444, Peirce’s emphasis)

In virtue of this, it is clear that, even if Peirce acknowledges the existence of important differences between logic and mathematics — for instance,
he remarks (CP 4.239) that while mathematics is the science which \textit{draws} necessary conclusions, logic is the science of \textit{drawing} necessary conclusions (Peirce’s emphasis) and that while mathematics is purely hypothetical, logic is categorical (CP 4.240), he also believes that both disciplines have a common basis —namely, both are grounded on icons which represent all their structural features:

The chief need for the Icons is in order to show the Forms of the synthesis of the elements of thought. For in precision of speech, Icons can represent nothing but Forms and Feelings. That is why Diagrams are indispensable in all Mathematics, from Vulgar Arithmetic and up, and in Logic are almost so. For Reasoning, nay, Logic generally, hinges entirely on Forms. (CP 4.544)

Thus, for Peirce, mathematical truths are logical truths in the sense that they are all built out of the same operations —namely, the observation and manipulation of certain icons. Because of this, mathematics is not fundamentally different from logic according to Peirce. They do have distinct aims and the nature of their assertions is different but, as Peirce (CP 3.560) emphasizes, ‘the true difference between the necessary logic of philosophy and mathematics is merely one of degree.’ Mathematical and logical truths have a common nature insofar as both are grounded on certain operations realized on pure icons and, considering that ‘a pure icon can convey no positive of factual information; for it affords no assurance that there is any such thing in nature’ (CP 4.447), they are all devoid of content. Consequently, it is not surprising that Peirce (CP 4.237) maintains that one of the most striking features of mathematics is ‘the fleshless and skeletal build of its propositions’. Indeed, if mathematics is not really different from logic (as Peirce maintains) and the laws of logic abstract from all content, mathematical propositions must also prescind from all content.

6 Conclusion

In this paper, I have provided a comparative study of the positions of Peirce and Frege regarding the nature of logic. I have shown that their views are virtually identical in many respects, with the exception of one crucial issue (namely, whether the laws of logic have content or not), and I have also shown how this difference can be used to cast light on the debate concerning whether Peirce endorses logicism or not. In particular, I have argued that he endorses logicism, but that his logicism is more similar to the version
endorsed by Wittgenstein in the *Tractatus* than to the versions endorsed by Frege and Russell.

These conclusions suggest a number of lines of future research: on one side, it would desirable to explain why Peirce and Frege agreed on so many issues given that they worked in virtual isolation of each other and, on the other side, given the similarities between the strategies used by Peirce and Wittgenstein to provide unified accounts of arithmetic/mathematics and logic, it would be desirable to determine whether Wittgenstein was directly or indirectly influenced by Peirce’s views.
References


