# Reverse Price Discrimination with Bayesian Buyers*

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**Abstract**

The paper studies price discrimination under the situation where buyers' prior valuations are initially observable to a seller but buyers receive further information about a product or service which remains private thereafter. The buyers interpret new information via Bayes' rule. We show that, in this environment, prices are not monotone in buyers' *prior* valuations. Interestingly, this results in the possibility that a seller intentionally offers a higher price to a low valuation buyer rather than a high valuation buyer (*Reverse Price Discrimination*), which sharply contrasts with the standard result of price discrimination. We derive this result in both monopoly and duopoly markets. (JEL: D4, D8)

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# 1 Introduction

When you open the door to a car dealership, a salesperson will welcome you and ask how you are. At this moment, the salesperson may try to figure out your willingness to pay by looking at your appearance such as your gender, race, and age, and further by asking some related questions from which he could infer your income and so forth. Then the salesperson may offer you a personalized price based on this observation. This looks like a typical third-degree price discrimination: a seller is able to observe a buyer's willingness to pay and charge a higher price to a high valuation buyer.

However, the seller's pricing can be different from the one in a standard (third-degree) price discrimination if buyers' valuations can change depending on private information they additionally gather. For example, car buyers can do test drives and home buyers can visit houses before their purchase. These advance activities allow consumers to learn more about the products in which they are interested. In this situation, although buyers' prior willingness to pay might be observable to a seller, their posterior valuations formed by the new information they acquire may remain private. A key feature is that buyers update their valuations in a Bayesian way in the sense that buyers' posterior valuations are systematically dependent on their prior beliefs through Bayes' rule.

The analysis of the optimal pricing in this situation is of particular interest because it begins by departing from the standard classification of price discrimination. As is well known, third-degree price discrimination is charging different prices based on some observable characteristics, whereas second-degree price discrimination considers the case where the buyers' types are not observable.[1] Our model considers the intermediate case in the following sense. Although a buyer's prior valuation is perfectly observable to the seller, the buyer receives new private information and updates his valuation before purchasing. As a consequence, the seller has to choose the optimal price based only on the buyer's prior valuation, but not on the posterior valuation. In this sense, we may refer to the pricing strategy in this particular situation as 2.5-degree price discrimination. Then what would be the optimal pricing rule for the seller who faces this environment? We address this question in both monopoly and duopoly cases.

In the monopoly case, we show that prices can be non-monotone in buyers' *prior* valuation. In other words, surprisingly, there exists the condition for which the seller offers a higher price to low valuation buyers than to high valuation buyers. Needless to say, this result stands in sharp contrast to the standard result of third-degree price discrimination where a higher (lower) price is charged to high (low) valuation buyers. Here is the intuition underlying this result.

When the seller makes a decision on the prices, she faces a traditional trade-off between getting a higher margin and getting a greater market share. This trade-off depends crucially on the price elasticity of demand, which is essentially determined by the interaction between a buyer's prior valuations and the precision of information available to the buyer. When information precision is not extreme (i.e., neither too accurate nor too vague), we find that the demand is elastic (inelastic) when the buyer's prior valuation is high (low). Thus, if the buyer's prior valuation is high, the seller

[1] There are the two excellent survey papers about price discrimination: Varian (1989) and Stole (2007).

chooses to offer a low price because the buyer's updated valuation does not decline significantly even when he receives unfavorable private information. On the other hand, if his prior valuation is low, it is more profitable for the seller to charge a high price and target only the buyer who receives favorable private information because the price under which the buyer with unfavorable information is covered is too low to be profitable. This leads to the result that a higher price can be offered to buyers with lower willingness to pay and vice versa.

We next explore a similar phenomenon in a duopoly market. Consider that the competing sellers share buyers' prior valuations and the buyers gather further private information about the product or service provided by each seller. Again, we show that reverse price discrimination can arise in this case as well. In the duopoly case, new information plays a role of differentiating two products. Especially when the buyer gathers opposite information about each product (for example, one is good and the other is bad), he perceives that two products are more differentiated than before and thus the price competition between the sellers can be mitigated. We find that the degree of differentiation becomes higher, and so does the equilibrium price, when the buyer has an intermediate prior valuation. The intuition is straightforward. The buyer with intermediate prior valuation is not certain whether the product would be a good match or a bad match for him, thus he is quite sensitive to the new information he gathers. Hence, when he receives opposite information from the two products he tends to be strongly biased toward the product with favorable information. This, in turn, gives the sellers a weaker incentive to attract the buyer through offering a lower price. For the opposite reason, the buyer with extreme (either high or low) prior valuation has the advantage of being offered a lower price, since he is likely to perceive that the two products are still similar even after updating his valuations. As a result, the equilibrium prices do not change monotonically with the buyer's prior valuation. This yields the possibility that the buyer with lower prior valuation is charged the higher price than the buyer with higher prior valuation.

Our main results are distinct from the standard result of price discrimination in that the seller charges a higher price to buyers with lower willingness to pay as a profitable strategy. In order to emphasize this contrast, we refer to this outcome as *reverse price discrimination*. This type of price discrimination can be prevalent, in particular, in a market for professional services such as the ones provided by lawyers, car mechanics, and home improvement contractors. Consumers in these markets are definitely uncertain of the quality of services, so they try to gather additional private information as much as possible. And the sellers in these markets often offer personalized prices to different individuals.

The result derived in our model may provide an alternative explanation to the discrimination in car sales observed by Ayres (1995) and Ayres and Siegelman (1995).[2] They tested how new-car dealerships quoted differently across customers' races. They found that dealers offered significantly lower prices to white buyers than to non-white buyers even though white people are believed to have a higher willingness to pay than non-white people, which is why price discrimination appears

[2] See Yinger (1998) for a good survey and summary of the discrimination literature in consumer markets.

to be the racial discrimination.[3] However, our model suggests the possibility that a salesperson may offer a higher price to the group which is more likely to have a lower willingness to pay because it is profitable to target only those who have favorable posterior impressions.

Our result can also explain reward programs for frequent customers. When a seller can identify repeated customers, she regards them as consumers with a high willingness to pay. The recent literature on purchase-based price discrimination has studied this issue extensively.[4] A common finding in this literature is the so called "ratchet effect", which describes price discrimination against loyal customers by charging a higher price. However, it is also not hard to find numerous opposite examples where sellers often use price discrimination favorable to loyal customers. For example, airline companies provide discounts and free upgrades to frequent customers. Automobile companies offer loyalty rebates to customers who currently own the same brand car. In our model, a buyer's prior valuation can be thought of as a brand loyalty which can be assessed by a seller through a customer's purchase history. Our model then provides reasoning about why a seller may offer discount prices to the loyal customers.

Price discrimination with incomplete information has been studied in the environment of screening or self-selection mechanism followed by Mussa and Rosen (1978) and Maskin and Riley (1984). In particular, Lewis and Sappington (1994) and Courty and Li (2000) are closely related to ours because they also study the environment where consumers are initially uncertain of their valuations that are in part revealed afterwards. However, while both papers study second-degree price discrimination in which firms are assumed not to be able to observe consumers' expected valuations, the seller can observe them and price discriminate based on them in our paper.[5] Above all, to the best of our knowledge, our paper is the first study of reverse price discrimination.

The rest of this article proceeds as follows. Section 2 describes the structure and assumptions of the model. In Section 3, we study the monopolist's price discrimination when a buyer has a unit demand and receives a binary signal. In Section 4, we show that the main result carries over to the duopoly case. In Section 5, we elaborate on extensions of the model. We show the robustness of our main result even after incorporating a non-unit demand function and general information structure. Section 6 concludes.

[3] It is the empirical fact that white people have higher average income than non-white people do. So it would not be add-hoc to say that white people are believed to have a higher willingness to pay.

[4] See Armstrong (2006) and Fudenberg and Villas-Boas (2005) for the review of the literature on behavior-based price discrimination.

[5] In this paper, we confine our attention to the case where the seller makes a take-it-or-leave-it offer after the buyer receives an informative signal. On the other hand, the seller may allow the buyer to choose the product and precision of signals before receiving the signals. This can be thought of as the second-degree price discrimination through providing different levels of information. This is the issue that our companion paper, Bang, Choi, and Kim (2010), studies.

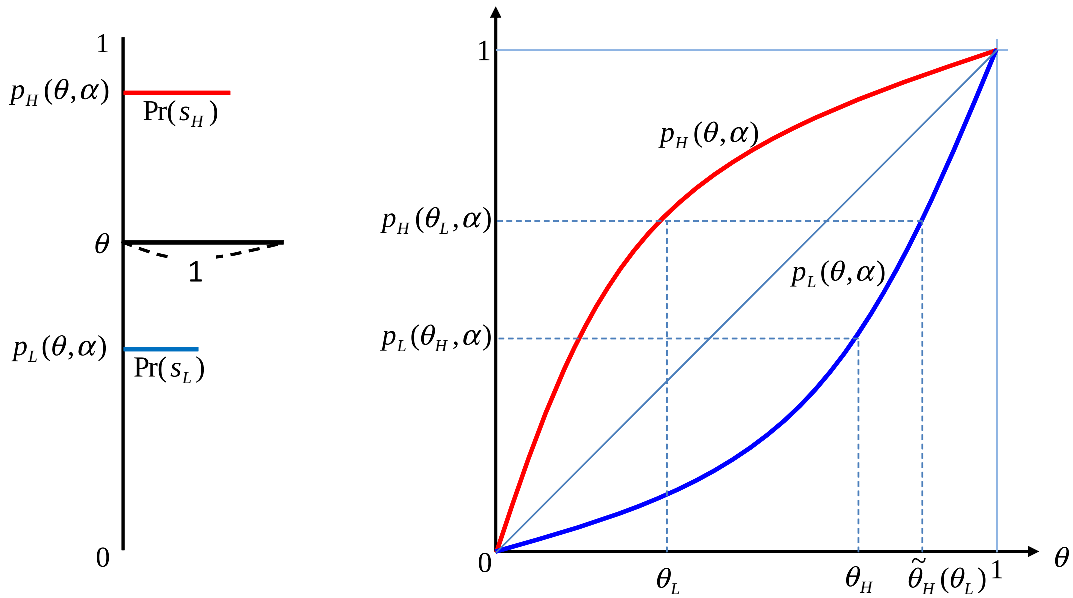


Figure 1: Demand

## 2 The Model

In this section, we explain the basic setup of the model which will be used throughout the paper. Based on this basic model, in subsequent sections, we consider both monopoly and duopoly cases with some extensions. The additional assumptions or setups needed in each scenario will be introduced as needed in each section.

**Players.** The buyers have a unit demand for the good, which is solely supplied by seller(s). The true value of the good depends on the match between a buyer's preference and the features of the good, which is denoted by $v \in \{H, L\}$. If $v = H$, the good is a good match with a buyer and if $v = L$, it is a bad match. A buyer's prior belief for $v = H$ is denoted as $\theta \in [0, 1]$. We normalize the buyer's valuation for the good to be 1 for $v = H$ and 0 for $v = L$. Accordingly, $\theta$ can be thought of as the buyer's *prior* valuation for the good and therefore as the willingness to pay. The buyer's type $\theta$ is public information and observable to the seller(s). For simplicity, the reservation value of seller(s) for the good is assumed to be 0.

**Information.** The buyers receive private information about the good while they are inspecting the good before purchase. Although the buyers are ex ante homogeneous, they may draw different binary signals, $s \in \{s_H, s_L\}$, on their match value. The realization of a signal is privately observed by the buyers, so it is private information. As is standard, the signals partially reveal the true match value of the good in the sense of Blackwell,

$$\begin{aligned}\Pr(s_H|v &= H) = \Pr(s_L|v = L) = \alpha \\ \Pr(s_L|v &= H) = \Pr(s_H|v = L) = 1 - \alpha\end{aligned}$$

where $\alpha \in \left(\frac{1}{2}, 1\right)$ without loss of generality. $\alpha$ is often interpreted as the precision or the quality of a signal and is common knowledge.

In addition, one interesting interpretation of $\alpha$ is related to the products classification. Nelson (1970) classifies products into two categories: search goods and experience goods. He defines a search good as one whose qualities can be easily evaluated by the consumer before purchase. Similarly, he defines an experience good as one whose qualities are difficult to observe before purchase. According to this definition, $\alpha = 1$ is comparable to a search good because the buyers are able to observe the quality of a good completely before their purchase. By contrast, $\alpha = 1/2$ represents an experience good because the buyers are not able to observe the quality in advance.[6] Thus, if $\alpha \in \left(\frac{1}{2}, 1\right)$, the good can be thought of as one which is neither perfectly a search good nor perfectly an experience good. In reality, there would be many goods and services which fall somewhere between those two extreme types of goods, and those are our main interest in this paper.

As the buyer receives a signal $s \in \{s_H, s_L\}$, he updates his beliefs on the match value for the good. Let us refer to $p_H(\theta, \alpha)$ and $p_L(\theta, \alpha)$ as the buyer's posterior valuation when the buyer with a prior $\theta \in (0, 1)$ draws a signal $s_H$ and $s_L$, respectively. Then, Bayes' rule leads to

$$p_H(\theta, \alpha) = \frac{\alpha\theta}{\alpha\theta + (1-\alpha)(1-\theta)} \text{ and} \quad (1)$$

$$p_L(\theta, \alpha) = \frac{(1-\alpha)\theta}{\alpha(1-\theta) + (1-\alpha)\theta} \quad (2)$$

because we normalize the buyer's valuation for the good to be 1 for $v = H$ and 0 for $v = L$.

The buyer's posterior valuation is a mean-preserving spread of his prior belief. A signal disperses the prior belief to two-point distribution: $p_H(\theta, \alpha)$ with the probability $\Pr(s_H)$ and $p_L(\theta, \alpha)$ with the probability $\Pr(s_L)$. The probabilities that the buyer receives a good signal and bad signal are given by $\Pr(s_H) = \sum_{v \in \{L,H\}} \Pr(s_H|v) \Pr(v)$ and $\Pr(s_L) = \sum_{v \in \{L,H\}} \Pr(s_L|v) \Pr(v)$, respectively. In this information structure, the following properties are well-known. First, the posterior valuations $p_H(\theta, \alpha)$ and $p_L(\theta, \alpha)$ are increasing in $\theta$ at a decreasing rate and at an increasing rate respectively:

$$\frac{\partial p_H(\theta, \alpha)}{\partial \theta} > 0, \frac{\partial p_L(\theta, \alpha)}{\partial \theta} > 0, \frac{\partial^2 p_H(\theta, \alpha)}{\partial \theta} < 0, \text{ and } \frac{\partial^2 p_L(\theta, \alpha)}{\partial \theta} > 0 \quad (3)$$

as represented in Figure 1. Second, $p_H(\theta, \alpha)$ is an increasing function and $p_L(\theta, \alpha)$ is a decreasing function in $\alpha$:

$$\frac{\partial p_H(\theta, \alpha)}{\partial \alpha} > 0 \text{ and } \frac{\partial p_L(\theta, \alpha)}{\partial \alpha} < 0 \quad (4)$$

[6] If $\alpha = \frac{1}{2}$, it means that the private signal $s$ does not provide any information about the true value.

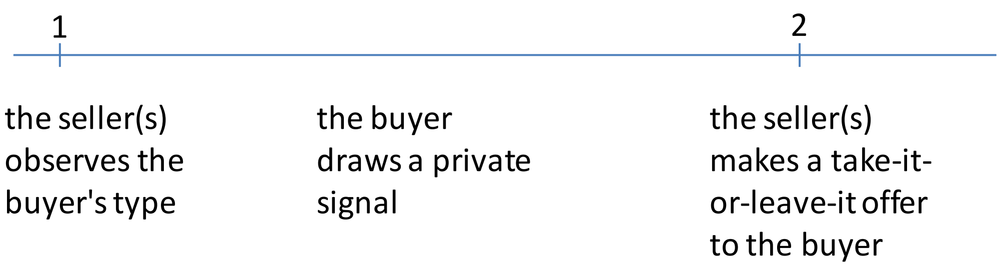


Figure 2: Timing

**Timing.** The timing of the game is summarized in Figure 2. In the first stage, the seller can observe the buyer's prior valuation $\theta$ perfectly. At the same time, the buyer draws a private signal about the match value of the good. In the second stage, the seller quotes price $P$ to the buyer and makes a take-it-or-leave-it offer. We assume that the buyer prefers to purchase the good when he is indifferent between buying and not buying the good.

## 3 Monopoly

Now we consider the case where there is only one seller. In this section, we assume that there are two types of buyers and type-$i$ buyer's prior valuation is $\theta_i \in \{\theta_L, \theta_H\}$ where $0 \leq \theta_L < \frac{1}{2} < \theta_H \leq 1$.[7] We define the buyer as a type-H if $\theta_i = \theta_H$ and a type-L if $\theta_i = \theta_L$.

Given the information structure, we find the optimal prices that the seller offers to each type of buyer. Note that when buyer $i$ receives a good signal, he will purchase the good if and only if $p_H(\theta_i, \alpha) - P_i \geq 0$. Likewise, when he receives a bad signal, he will purchase if and only if $p_L(\theta_i, \alpha) - P_i \geq 0$. Hence, the expected demand is the equilibrium probability that buyer $i$ accepts an offer for a given price $P_i$, which is given by

$$D(P_i) = \begin{cases} 1, & \text{if } P_i \leq p_L(\theta_i, \alpha), \\ \Pr(s_H) = \alpha\theta_i + (1-\alpha)(1-\theta_i), & \text{if } p_L(\theta_i, \alpha) < P_i \leq p_H(\theta_i, \alpha), \\ 0, & \text{if } P_i > p_H(\theta_i, \alpha). \end{cases}$$

Figure 1 illustrates the demand function depending on the realization of a private signal.

For each type of buyer, the seller has to choose between two alternative prices: (i) a high price at which only the buyer who receives a good signal $s = s_H$ can afford to buy, i.e., $P_i = p_H(\theta_i, \alpha)$ and (ii) a low price at which everyone including the buyer who receives a bad signal $s = s_L$ can afford to buy, i.e., $P_i = p_L(\theta_i, \alpha)$. When a seller charges $p_H(\theta_i, \alpha)$ to a type-$i$ buyer, she can sell

[7] This can also be interpreted as follows: a seller regards the buyer with $\theta \in \left[0, \frac{1}{2}\right)$ as a type-L and the one with $\theta \in \left(\frac{1}{2}, 1\right]$ as a type-H. This assumption simplifies our analysis without a significant change in our main results.

the good with probability $\Pr(s_H)$ which is the probability that the buyer receives a signal $s = s_H$. On the other hand, she can sell the good with probability 1 if she charges $p_L(\theta_i, \alpha)$. Keep in mind that the seller can observe the buyer's type ($\theta_i$, *prior* valuation) and therefore can offer different prices to two different types of buyers. As in any model of price discrimination, we rule out a resale possibility.

Let $\pi_H(\theta_i, \alpha)$ and $\pi_L(\theta_i, \alpha)$ be the seller's expected profit when she charges $p_H(\theta_i, \alpha)$ and $p_L(\theta_i, \alpha)$ to a type-$i$ buyer. Then, we obtain from (1) and (2)

$$\pi_H(\theta_i, \alpha) = p_H(\theta_i, \alpha) \Pr(s_H) = \alpha\theta_i \text{ and} \tag{5}$$

$$\pi_L(\theta_i, \alpha) = p_L(\theta_i, \alpha) \cdot 1 = \frac{(1-\alpha)\theta_i}{\alpha(1-\theta_i) + (1-\alpha)\theta_i}. \tag{6}$$

Comparing $\pi_H(\theta_i, \alpha)$ and $\pi_L(\theta_i, \alpha)$ determines the optimal price the seller charges to type-$i$ buyer. In fact, the comparison tells us whether the expected demand is inelastic, unit-elastic, or elastic. Since the expected demand is a two-point distribution, we shall use the midpoint method for calculating the price elasticity of demand.

$$\varepsilon_p = -\frac{\Pr(s_H) - 1}{p_H(\theta_i, \alpha) - p_L(\theta_i, \alpha)} \cdot \frac{\frac{p_H(\theta_i,\alpha)+p_L(\theta_i,\alpha)}{2}}{\frac{1+\Pr(s_H)}{2}}$$

**Lemma 1** $\pi_H(\theta_i, \alpha) \gtreqless \pi_L(\theta_i, \alpha)$ *as* $\varepsilon_p \lesseqgtr 1$. *The seller charges a higher price,* $p_H(\theta_i, \alpha)$, *when the demand is inelastic and a lower price,* $p_L(\theta_i, \alpha)$, *when the demand is elastic.*

The tension in determining the price is the trade-off between getting a higher margin by charging $p_H(\theta_i, \alpha)$ and getting a greater market share by charging $p_L(\theta_i, \alpha)$. The seller's pricing decision is determined by the price elasticity of demand. Now, let us show that the elasticity systematically depends on the interaction between a buyer's prior valuation and the precision of information. There are three possible cases. The following proposition summarizes the result.

**Proposition 1** *Let* $\alpha_1 = \alpha^*(\theta_i = \theta_L)$ *and* $\alpha_2 = \alpha^*(\theta_i = \theta_H)$ *where*

$$\alpha^*(\theta_i) = \frac{\left((\theta_i + 1) - \sqrt{\theta_i^2 - 6\theta_i + 5}\right)}{2(2\theta_i - 1)}.$$

*Then, given* $\theta_H > 1/2 > \theta_L$, *there exist* $\alpha_1, \alpha_2 \in \left(\frac{1}{2}, 1\right)$ *such that the following results hold.*

*(i) If* $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, *a seller charges* $p_L(\theta_i, \alpha)$ *for each type* $i$.
*(ii) If* $\alpha \in [\alpha_1, \alpha_2]$, *a seller charges* $p_H(\theta_L, \alpha)$ *for a type-L buyer and* $p_L(\theta_H, \alpha)$ *for a type-H buyer.*
*(iii) If* $\alpha \in (\alpha_2, 1)$, *a seller charges* $p_H(\theta_i, \alpha)$ *for each type* $i$.

**Proof of Proposition 1**

*In the appendix.*

If information is sufficiently precise (case iii), the demand becomes inelastic. In this case, the seller prefers getting a higher margin to getting a greater market share because she has to lower the price too much in order to sell to the buyer with a bad signal. In other words, the price $p_H(\theta_i, \alpha)$ is high enough to compensate the loss in profit from losing the buyers with a bad signal. Hence the seller charges $p_H(\theta_i, \alpha)$ to each type of buyer. By contrast, if information is too vague (case i), the demand is elastic, so the seller prefers to have the large market share and serve all buyers by charging $p_L(\theta_i, \alpha)$. In these two cases, the elasticity is solely determined by the quality of information.

The most interesting case is the one where information quality is intermediate and the buyer's prior valuation determines the elasticity (case ii). When a buyer is type-H, it is more profitable to increase a market share because a type-H buyer's posterior valuation even with $s = s_L$ is relatively high enough to make the seller offer $p_L(\theta_L, \alpha)$ and serve all type-H buyers. On the other hand, when a buyer is type-L, the price for serving the buyers with $s = s_L$ should be significantly low. Hence, increasing a market share is not attractive, so the seller charges the price $p_H(\theta_L, \alpha)$ to type-L buyers. Now we compare the prices offered to each type of buyer.

**Proposition 2** *If $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$ or $\alpha \in (\alpha_2, 1)$, the price offered to a type-H buyer is higher than that offered to a type-L buyer.*

If information quality is sufficiently precise, i.e., $\alpha \in (\alpha_2, 1)$, the seller charges $p_H(\theta_i, \alpha)$ to each type $i$. Since $p_H(\theta_i, \alpha)$ is an increasing function in $\theta_i$ from (1), we obtain $p_H(\theta_H, \alpha) > p_H(\theta_L, \alpha)$. On the other hand, if information quality is sufficiently imprecise, i.e., $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, the seller charges $p_L(\theta_i, \alpha)$ for each type $i$. Again, we obtain $p_L(\theta_H, \alpha) > p_L(\theta_L, \alpha)$ because $p_L(\theta_i, \alpha)$ is also an increasing function in $\theta_i$ from (2). Hence, if information quality is either sufficiently precise or imprecise, the price offered to a type-H buyer should be higher than that for a type-L buyer. This implies that the standard result of price discrimination holds if the goods are close to search goods or experience goods.

Now, the striking result is that we find the possibility of *reverse price discrimination* for the case of in-between goods where $\alpha \in [\alpha_1, \alpha_2]$. The price offered to a type-L buyer can be higher than that offered to a type-H buyer. Since $p_H(\theta_i, \alpha)$ is concave in $\theta_i$ and $p_L(\theta_i, \alpha)$ is convex in $\theta_i$, we can easily denote the case in which $p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha)$ by looking at a certain $\theta_L$ and $\theta_H$, as shown in Figure 1. We summarize this result more formally in the following proposition.

**Proposition 3** *Suppose that $\alpha \in [\alpha_1, \alpha_2]$. Let us define $\widetilde{\theta}_H(\theta_L)$ such that $p_L(\widetilde{\theta}_H(\theta_L), \alpha) = p_H(\theta_L, \alpha)$ for given $\theta_L$ and $\widetilde{\theta}_L(\theta_H)$ such that $p_H(\widetilde{\theta}_L(\theta_H), \alpha) = p_L(\theta_H, \alpha)$ for given $\theta_H$. We obtain the reverse* *price discrimination,*

$$p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha),$$

*if* $\theta_H < \widetilde{\theta}_H(\theta_L) = \frac{\alpha^2\theta_L}{\alpha^2\theta_L + (1-\alpha)^2(1-\theta_L)}$, *or equivalently* $\widetilde{\theta}_L(\theta_H) = \frac{(\alpha-1)^2\theta_H}{(\theta_H - 2\alpha\theta_H + \alpha^2)} < \theta_L$.

**Proof of Proposition 3**

*In the appendix.*

According to Proposition 3, for the reverse price discrimination to arise, the difference between two prior valuations $\theta_L$ and $\theta_H$ should be bounded above. Also note that $\frac{\partial(\widetilde{\theta}_H(\theta_L))}{\partial\alpha} = \frac{2(\theta_L-1)(\alpha-1)\alpha\theta_L}{(2\alpha\theta_L - \theta_L - 2\alpha + \alpha^2 + 1)^2} > 0$ and $\frac{\partial(\widetilde{\theta}_L(\theta_H))}{\partial\alpha} = \frac{2(1-\theta_H)(\alpha-1)\alpha\theta_H}{(2\alpha\theta_H - \theta_H - \alpha^2)^2} < 0$. That is, for given $\theta_L$, as $\alpha$ increases, the value of $\widetilde{\theta}_H(\theta_L)$ below which $p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha)$ increases. Also for given $\theta_H$, as $\alpha$ increases, the value of $\widetilde{\theta}_L(\theta_H)$ above which $p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha)$ decreases. These imply that as information quality $\alpha$ increases (decreases), the parameter set of $\theta_L$ and $\theta_H$ for which the reverse price discrimination is derived becomes larger (smaller). Here is the reason. As information quality increases, the type-L buyer's posterior valuation, after observing a good signal, becomes relatively high enough to compensate the initial low prior belief. Also, although the type-H buyer's initial prior belief is high, his posterior belief after observing a bad signal becomes relatively low. These mean that the curvatures of both functions $p_H(\theta_L, \alpha)$ and $p_L(\theta_H, \alpha)$ become larger as $\alpha$ increases, as seen in Figure 1. Hence, even though the prior valuations differ much, as information quality increases, the reverse price discrimination is more likely to arise.

We believe that our result may provide a new explanation for Ayres and Siegelman (1995). According to their findings, the car dealers offered the $\$1,061$ higher price to non-white buyers rather than to white buyers, although the non-white buyers are believed to have a lower willingness to pay than white buyers.[8] Our model suggests that their findings can be due to the dealer's strategy, which is to target only the buyers having a higher posterior valuation in the case of non-white buyers. Provided that their initial willingness to pay, from a low prior valuation, is relatively low, the dealer has to lower the price significantly to serve everyone including the ones with low posterior valuation. Thus, it may yield a larger profit for the seller to target only the ones with high posterior valuation. On the other hand, even though the white buyers are believed to have relatively high willingness to pay, the seller may want to offer a relatively low price in order to entice all the buyers regardless of the signals they receive, since even the ones getting a bad signal still have a relatively high posterior valuation.[9]

[8] Ayres and Siegelman (1995) discuss the following three hypotheses: i) More people in minority groups are not aware of the fact that the sticker price is negotiable. ii) They are more likely to be averse to conducting negotiations. iii) Black Americans might have higher willingness to pay in terms of search costs. Especially, iii) is directly related to price discrimination based on the difference in consumers' willingness to pay. As to this, our hypothesis is that salespeople may charge a higher price to black Americans even though they are more likely to have lower willingness to pay.

[9] We do not claim that our reasoning is the most proper explanation for the finding in Ayres and Siegelman (1995). We believe that price discrimination in car sales can stem from a complex interaction of all factors discussed in Ayres and Siegelman (1995) and our complementary reasoning.

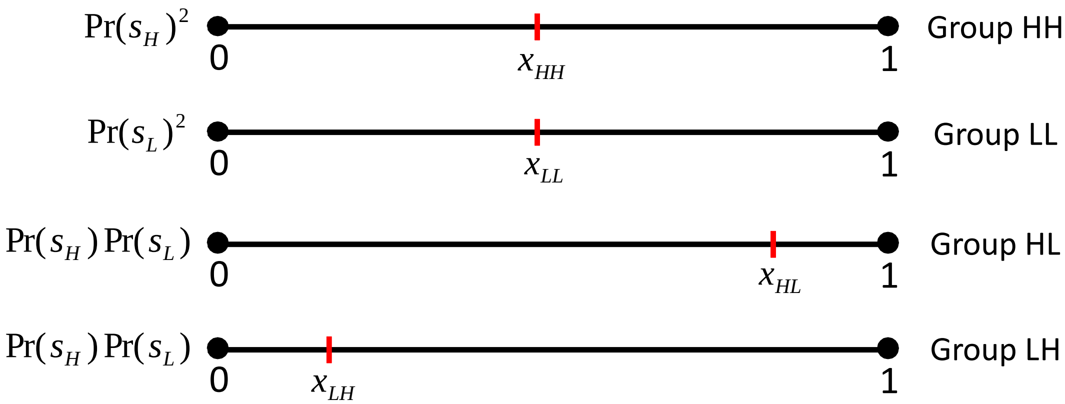


Figure 3: Demand in the Hotelling model

# 4 Duopoly: Hotelling Model

We have shown that reverse price discrimination can be derived in the monopoly case. Then, can it also be the case even when sellers compete with each other? In this section, we extend our analysis to a duopoly market and consider the case where two sellers are located at two end points on the Hotelling line of unit length. We also show that the equilibrium prices can be non-monotone in the buyer's prior valuation in the duopoly case.

Two sellers, $A$ and $B$, supply differentiated products $A$ and $B$ respectively. As in a standard Hotelling model, each buyer is uniformly distributed and indexed as $x \in [0, 1]$ which denotes the buyer's location or brand preference. Buyers purchase either one unit of a good from only one seller or nothing. Type $(\theta, x)$ buyer's value is $w + \theta - tx$ for good $A$ and $w + \theta - t(1 - x)$ for good $B$. Note that $w + \theta$ is the intrinsic value of consuming the product. While $w$ captures the product value with no uncertainty, $\theta$ represents the value with uncertainty. We assume that $w$ is sufficiently large so that the market is fully covered. The parameter $t > 0$ reflects the degree of product differentiation. To keep consistency with the monopoly case, we assume that the buyers' prior valuations are public information and known to sellers. For analytical simplicity, we assume that the symmetric information structure is exogenously given to the sellers.

Now the buyers independently receive a private signal from each seller. They face one of four possible cases according to the combinations of signal realizations $\left(s^A, s^B\right)$, where $s^A, s^B \in \{s_H, s_L\}$ denote the signals from sellers A and B. As both signals $s^A$ and $s^B$ are buyers' private information, each seller expects the possible outcome as follows: (i) $\left(s^A, s^B\right) = (s_H, s_H)$ with probability $\Pr(s_H)^2$, (ii) $\left(s^A, s^B\right) = (s_L, s_L)$ with probability $\Pr(s_L)^2$, (iii) $\left(s^A, s^B\right) = (s_H, s_L)$ with probability $\Pr(s_H)\Pr(s_L)$, and (iv) $\left(s^A, s^B\right) = (s_L, s_H)$ with probability $\Pr(s_H)\Pr(s_L)$.

Figure 3 illustrates the demand structure as a result of the realization of private signals. If the buyers receive the same signals from both sellers, i.e., $\left(s^A, s^B\right) = (s_H, s_H)$ or $(s_L, s_L)$, the private signals do not provide additional information about which good would be more preferable than the other. As their relative preference between two goods does not change, there is no difference from the standard Hotelling model in this case. Hence the location of marginal buyers who are indifferent between the two goods is denoted by

$$x_{HH} = x_{LL} = \frac{1}{2} + \frac{P^B - P^A}{2t} \tag{7}$$

where the subscription for $x$ denotes the group as illustrated in Figure 3, and $P^A$ and $P^B$ are the prices offered by sellers $A$ and $B$, respectively.[10]

On the other hand, if the different signals are drawn from two sellers, the marginal buyers for the case where $\left(s^A, s^B\right) = (s_H, s_L)$ and $(s_L, s_H)$ are, respectively,

$$x_{HL} = \frac{1}{2} + \frac{P^B - P^A + \Delta(\theta,\alpha)}{2t} \text{ and } x_{LH} = \frac{1}{2} + \frac{P^B - P^A - \Delta(\theta,\alpha)}{2t} \tag{8}$$

where $\Delta(\theta,\alpha) \equiv p_H(\theta,\alpha) - p_L(\theta,\alpha)$. Here, $\Delta(\theta,\alpha)$ represents a buyer's bias after observing different signals. Note that depending on the size of the bias, the marginal buyers may not exist in the Hotelling line. That is, $x_{HL}$ can be greater than 1 and/or $x_{LH}$ can be smaller than 0.

**Lemma 2** *(1) If* $\Delta(\theta,\alpha) < t$*, then* $0 < x_{HL} < 1$ *and* $0 < x_{LH} < 1$*. (2) If* $\Delta(\theta,\alpha) \geq t$*, then* $x_{HL} \geq 1$ *and* $x_{LH} \leq 0$*.*

**Proof of Lemma 2**

*In the appendix.*

When $\Delta(\theta,\alpha) < t$, seller $A'$s demand function is written by

$$D_A = (\Pr(s_H)^2 + \Pr(s_L)^2)x_{HH} + \Pr(s_H)\Pr(s_L)(x_{HL} + x_{LH}).$$

In this case, since the bias is small, the marginal buyers exist and stand on the interior point in the Hotelling line. By contrast, when $\Delta(\theta,\alpha) \geq t$, seller $A'$s demand function is given by

$$D_A = (\Pr(s_H)^2 + \Pr(s_L)^2)x_{HH} + \Pr(s_H)\Pr(s_L).$$

Now, the bias is large enough relative to the price difference, so the two sellers can avoid competition for the two groups of buyers, groups HL and LH. That is, all buyers in group HL purchase good $A$, while all buyers in group LH purchase good $B$. We can write down seller $B'$s demand function in a similar way. Then, for $j \in \{A, B\}$, each seller $j$'s demand function can be derived as follows.

[10] We only consider the interior solution such as $0 < x_{HH}, x_{LL} < 1$. This implies that the prices difference is less than the product differentiation. For example, if $P^A - P^B > t$, $\frac{1}{2} + \frac{P^B - P^A}{2t} < 0$ and if $P^B - P^A > t$, $\frac{1}{2} + \frac{P^B - P^A}{2t} > 1$. That is, we exclude the possibility that seller $j \in \{A, B\}$ dominates the market of group HH and LL only due to the price effects such that $P^j$ is far lower than $P^{-j}$. This also implies that we focus on the symmetric equilibrium although there may exist the asymmetric cases where $0 < x_{HL} < 1$ and $x_{LH} < 0$ and $x_{HL} > 1$ and $0 < x_{LH} < 1$.

$$D_j = \begin{cases} \frac{t+P^{-j}-P^j}{2t}, & \text{if } \Delta(\theta,\alpha) < t, \\ (\Pr(s_H)^2+\Pr(s_L)^2)\left(\frac{1}{2}+\frac{P^{-j}-P^j}{2t}\right)+\Pr(s_H)\Pr(s_L), & \text{if } \Delta(\theta,\alpha) \geq t. \end{cases}$$

In turn, solving the two sellers' maximization problems, the symmetric equilibrium prices can be readily shown as follows.

**Lemma 3** *In the symmetric equilibrium, each seller's optimal price is as follows.*

$$P^*(\theta) = P^{A*} = P^{B*} = \begin{cases} t, & \text{if } \Delta(\theta,\alpha) < t, \\ t\left(\frac{(\Pr(s_H)+\Pr(s_L))^2}{(\Pr(s_H)^2+\Pr(s_L)^2)}\right), & \text{if } \Delta(\theta,\alpha) \geq t, \end{cases}$$

*where* $\Pr(s_H) = \alpha\theta + (1-\alpha)(1-\theta)$ *and* $\Pr(s_L) = \alpha(1-\theta) + (1-\alpha)\theta$.

**Proof of Lemma 3**

*In the appendix.*

When the bias is less than the product differentiation parameter $t$, the equilibrium prices are $t$. However, when the bias is greater than $t$, the equilibrium prices become greater than $t$. Now, let us describe the condition, $\Delta(\theta,\alpha) \gtreqless t$, as a function of $\theta$ in detail in order to show the relationship between the buyer's prior valuation and the equilibrium prices.

**Proposition 4** *(1) For* $t > 1$, $P^{A*} = P^{B*} = t$ *for all* $\theta \in (0,1)$. *(2) For* $0 < t < 1$, *if* $\alpha \in \left(\frac{1}{2}, \frac{t+1}{2}\right)$, $P^{A*} = P^{B*} = t$ *for all* $\theta \in (0,1)$. *On the other hand, if* $\alpha \in \left(\frac{t+1}{2}, 1\right)$, *there exist* $\underline{\theta} \in \left(0, \frac{1}{2}\right)$ *and* $\bar{\theta} \in \left(\frac{1}{2}, 1\right)$ *such that if* $\theta \in (0, \underline{\theta})$ *or* $\theta \in (\bar{\theta}, 1)$, $P^{A*} = P^{B*} = t$ *and if* $\theta \in [\underline{\theta}, \bar{\theta}]$, $P^{A*} = P^{B*} = \left(\frac{(\Pr(s_H)+\Pr(s_L))^2}{\Pr(s_H)^2+\Pr(s_L)^2}\right) t$.

**Proof of Proposition 4**

*In the appendix.*

When $t > 1$, the bias is always less than the product differentiation due to $\Delta(\theta,\alpha) = p_H(\theta,\alpha) - p_L(\theta,\alpha) < 1$. Thus, the equilibrium price is $t$ as in the standard Hotelling model. The interesting case is the one where $0 < t < 1$, in which the bias can be either greater or less than $t$. In this case, the information quality $\alpha$ matters in determining the equilibrium. If the information quality is relatively low, i.e., $\alpha \in \left(\frac{1}{2}, \frac{t+1}{2}\right)$, its effect is insignificant and the equilibrium price is still $P^{A*} = P^{B*} = t$. On the other hand, if the information quality is relatively high, i.e., $\alpha \in \left(\frac{t+1}{2}, 1\right)$, the private signal may have a dramatic effect on the buyer's posterior valuation, especially when the prior valuation is intermediate, thereby leading to changes in the equilibrium prices.

In the case that buyers have extreme $\theta$ (i.e., $\theta \in (0, \underline{\theta})$ or $\theta \in (\bar{\theta}, 1)$), they are stubborn about their prior valuation no matter what signals they draw. Thus, they hardly become inclined toward one seller over the other even if they receive different signals from the sellers. Because of this

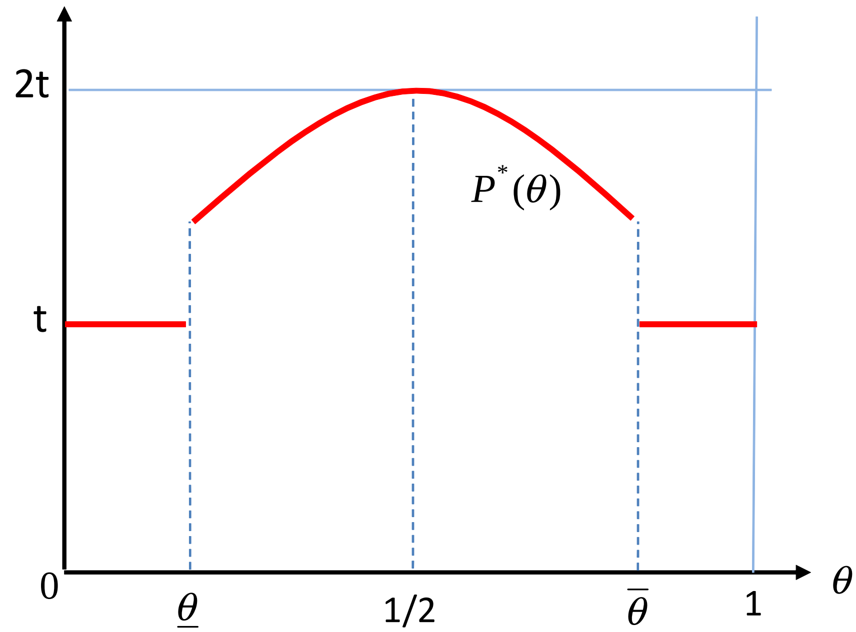


Figure 4: Equilibrium prices in duopoly when $0 < t < 1$ and $\alpha \in \left(\frac{t+1}{2}, 1\right)$.

insensitivity to the signals, the equilibrium prices are still determined to be $t$, as in the standard Hotelling model. On the other hand, in the case that buyers have intermediate $\theta$ (i.e., $\theta \in [\underline{\theta}, \bar{\theta}]$), new information starts to play a role in buyer's relative preferences. Note that these buyers can be regarded as not so obstinate about their prior beliefs on the match value of the product: the probability that their match value turns out to be either $v = H$ or $v = L$ is similar. Therefore, they are relatively sensitive to the new information, and are easily swayed by the signals they receive.

In particular, we find that the equilibrium prices are increasing in $\theta \in \left(\underline{\theta}, \frac{1}{2}\right)$, decreasing in $\theta \in \left(\frac{1}{2}, \bar{\theta}\right)$, and maximized at $\theta = \frac{1}{2}$. The intuition to understand this is simple. Recall that the bias $\Delta(\theta, \alpha) = p_H(\theta, \alpha) - p_L(\theta, \alpha)$ is increasing in $\theta \in \left(\underline{\theta}, \frac{1}{2}\right)$ and decreasing in $\theta \in \left(\frac{1}{2}, \bar{\theta}\right)$. This implies that buyers become signal-sensitive as the prior valuation $\theta$ is closer to 1/2. As the buyers are more (less) signal-sensitive, they become less (more) sensitive to the price differential, which, after all, mitigates (intensifies) price competition between the sellers. Hence the equilibrium prices are higher as $\theta$ is closer to 1/2. To put it simply, the easier consumers are swayed by the signals, the more they are exploited by the firms.

As a result, the equilibrium prices are not monotone with respect to $\theta$ as in Figure 4. This non-monotonicity suggests that, even in the duopoly market, there is a possibility that sellers offer a higher price to the buyer with lower willingness to pay than to the one with higher willingness to pay. With slight abuse of notation, we reuse the notations $\theta_H$ and $\theta_L$ to show the reverse price discrimination explicitly.

**Proposition 5** *Suppose that* $0 < t < 1$ *and* $\alpha \in \left(\frac{t+1}{2}, 1\right)$. *Then we obtain the reverse price* *discrimination,*

$$P^*(\theta_L) > P^*(\theta_H) \text{ for } \theta_L < \theta_H,$$

*if (1)* $\theta_L \in \left(\underline{\theta}, \frac{1}{2}\right)$ *and* $\theta_H \in (1-\theta_L, 1)$ *or (2)* $\theta_L \in \left(\frac{1}{2}, \bar{\theta}\right)$ *and* $\theta_H \in (\theta_L, 1)$.

Our paper may provide complementary but sharply contrary results to Armstrong (2006). He shows that it has no effect on the firms' prices and profits in the Hotelling model even though the firms can observe a consumer's valuation and target a personalized price to the consumers. In our setting with private information, however, the sellers may offer different personalized prices based on buyers' prior valuations.[11]

# 5 Extensions

In this section, we come back to the monopoly case and relax two key assumptions underlying our model, unit demand and binary signals, to check the robustness of the main results. In each case, we retain the other assumption to isolate the effect of each assumption.

## 5.1 Non-unit Demand: Linear Demand Case

We first relax the buyer's unit demand to a linear demand function. A buyer's demand function is given by $D(P) = \theta - P$ where $\theta$ is the buyer's prior valuation about the good. Depending on what signal $s \in \{s_H, s_L\}$ the buyer draws, his posterior demand function is either $D(P) = p_H(\theta, \alpha) - P$ or $D(P) = p_L(\theta, \alpha) - P$. As $s \in \{s_H, s_L\}$ is the buyer's private information, a seller does not know the actual demand and therefore should calculate the expected demand. If $P \leq p_L(\theta, \alpha)$, the expected demand is $D(P) = \Pr(s_H)\,(p_H(\theta, \alpha) - P) + \Pr(s_L)\,(p_L(\theta, \alpha) - P)$, which turns out to be $D(P) = \theta - P$. On the other hand, if $P > p_L(\theta, \alpha)$, the buyer with a bad signal does not want to purchase and thus the seller faces $D(P) = p_H(\theta, \alpha) - P$ with probability $\Pr(s_H)$. Then, the seller's expected demand with the Bayesian buyer is inwardly kinked as shown in Figure 5 (a).

$$D(P) = \begin{cases} \theta - P, & \text{if } P \leq p_L(\theta, \alpha), \\ \Pr(s_H)\,(p_H(\theta, \alpha) - P)\,, & \text{if } P \in (p_L(\theta, \alpha), p_H(\theta, \alpha)], \\ 0, & \text{if } P > p_H(\theta, \alpha). \end{cases}$$

[11] Damiano and Li (2007) study very similar issues such as price competition for privately informed buyers. A crucial difference is that they focus on the case in which the prior valuation is fixed as $\theta = 1/2$. In other words, price discrimination is not an issue in their paper. On the other hand, our focus is to find what prices two sellers offer to buyers based on $\theta$. In addition, it is worthwhile to explain the difference of the modeling strategy between the two papers. In their model, the two goods are *ex ante* identical. Thus, there is no pure-strategy Nash equilibrium, and it is hard to characterize and compare the equilibrium prices with our general prior $\theta$. This is the reason why we model the duopoly market by using the Hotelling model of product differentiation.

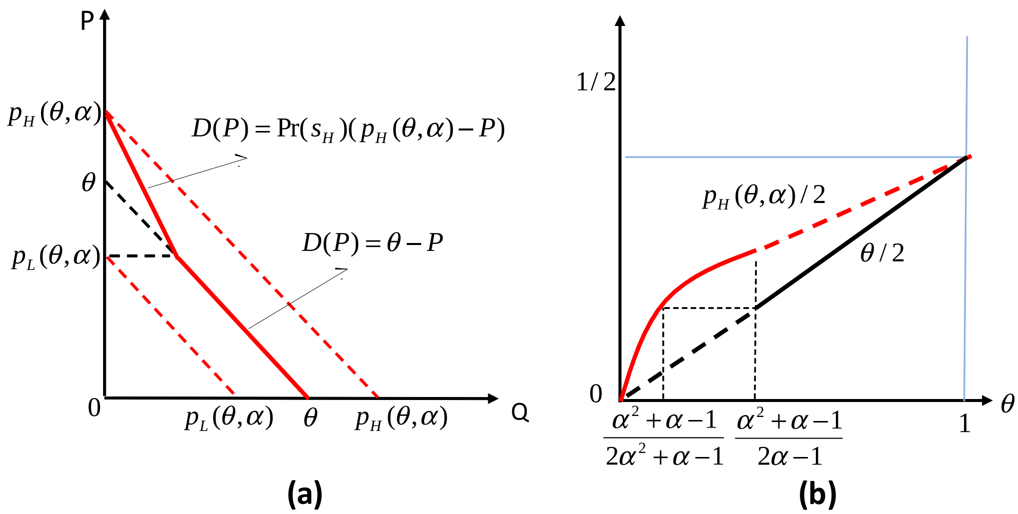


Figure 5: (a) Linear demand function (b) The optimal price when $\alpha \in (\frac{\sqrt{5}-1}{2}, 1)$

Since we assume zero production cost, the optimal price should be either $P^* = p_H(\theta, \alpha)/2$ or $\theta/2$. With a linear demand curve, the monopoly price is simply half of the price intercept. The analysis yields that the optimal price $P^*$ depends on the value of $\alpha$ and $\theta$ as follows.

**Lemma 4** *(1) Suppose that* $\alpha \in \left(\frac{1}{2}, \frac{\sqrt{5}-1}{2}\right)$*. Then,* $P^* = \theta/2$*. (2) Suppose that* $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$*. If* $\theta \in \left(0, \frac{\alpha^2+\alpha-1}{2\alpha-1}\right)$*,* $P^* = p_H(\theta, \alpha)/2$ *and if* $\theta \in \left(\frac{\alpha^2+\alpha-1}{2\alpha-1}, 1\right)$*,* $P^* = \frac{\theta}{2}$*.*

**Proof of Lemma 4**

*In the appendix.*

If the information quality is relatively low, i.e., $\alpha \in \left(\frac{1}{2}, \frac{\sqrt{5}-1}{2}\right)$, the optimal price is a monotone increasing function of $\theta$. On the other hand, if the information quality is relatively high, i.e., $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$, there exist two optimal prices contingent on the prior valuation $\theta$. The seller's decision is whether to charge a low price, $\theta/2$, for the sake of obtaining a high demand or charge a high price, $p_H(\theta, \alpha)/2$, for the sake of earning a high margin.

Keep in mind (4): $p_H(\theta, \alpha)$ is increasing in $\alpha$ and $p_L(\theta, \alpha)$ is decreasing in $\alpha$. When $\alpha$ is relatively low, the demand with a bad (good) signal is not much lower (higher). In this case, charging the low price for serving a buyer with a bad signal is more profitable. On the other hand, when $\alpha$ is relatively high, the demand with a bad (good) signal becomes low (high) enough so that sometimes charging a high price can be more profitable. More precisely, charging a high price is optimal if $\theta$ is relatively low. Otherwise, with a high $\theta$, the demand with a bad signal is not likely to be low enough for the high price to be optimal.[12]

[12] If $\alpha$ is sufficiently high, we need very high prior valuation in order to sustain this low optimal price.

Figure 5 (b) demonstrates the optimal price $P^*$, when $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$, as a function of $\theta$. There is a downward jump in the optimal price at a certain threshold. As a result, the optimal price is not monotone with respect to $\theta$. Then we can show the parameter set of $\theta$ for which the reverse price discrimination is obtained.

**Proposition 6** *Suppose that* $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$. *If* $\theta_L \in \left(\frac{\alpha+\alpha^2-1}{\alpha+2\alpha^2-1}, \frac{\alpha+\alpha^2-1}{2\alpha-1}\right)$ *and* $\theta_H \in \left(\frac{\alpha+\alpha^2-1}{2\alpha-1}, \dot{\theta}\right)$ *where* $\dot{\theta} = \frac{\alpha\theta_L}{(2\theta_L\alpha-\alpha-\theta_L+1)}$, *then* $P^*(\theta_L) > P^*(\theta_H)$ *for* $\theta_L < \theta_H$.

## 5.2 General Information Structure

Now, we incorporate a general information structure. The buyer observes a signal $s \in [0,1]$, distributed according to density $f_H(s)$ if $v = H$ and $f_L(s)$ if $v = L$. Both densities are bounded away from zero and we assume the monotone likelihood ratio property such that the likelihood function $f_L(s)/f_H(s)$ is decreasing in $s$. Then, the buyer's posterior valuation, given prior $\theta$, can be written by

$$p(\theta, s) = \frac{\theta f_H(s)}{\theta f_H(s) + (1-\theta) f_L(s)} = 1 \Bigg/ \left[1 + \frac{(1-\theta)}{\theta}\frac{f_L(s)}{f_H(s)}\right].$$

Given his prior belief and a signal about the quality, the buyer will make a decision on whether to buy or not. The buyer decides to purchase the good if he receives a signal better than his standard. Since the buyer's expected net payoff from buying the good is $p(\theta, s) - P$, he purchases if and only if $p(\theta, s) \geq P$, i.e., $\frac{f_L(s)}{f_H(s)} \leq \frac{\theta}{(1-\theta)}\frac{1-P}{P}$. The cutoff signal is defined as

$$\widehat{s}(\theta, P) \equiv \min\left\{ s \in [0,1] \,\middle|\, \frac{f_L(s)}{f_H(s)} \leq \frac{\theta}{(1-\theta)}\frac{1-P}{P} \right\}. \tag{9}$$

If the buyer receives a signal better (worse) than the standard $\widehat{s}(\theta, P)$, he decides to buy (not buy) the good or service. $\widehat{s}(\theta, P)$ is downward sloping in $\theta$, i.e., the buyer with a more optimistic belief will set a lower standard because $p(\theta, s)$ is increasing in both $\theta$ and $s$. Obviously, the buyer sets a higher standard for a higher price.

We turn to the seller's problem, where she chooses the price $P$ to maximize her profits. Since the buyer purchases the good when he receives a signal $s \geq \widehat{s}$, the expected demand can be written by

$$D(\widehat{s}) = \theta[1 - F_H(\widehat{s})] + (1-\theta)[1 - F_L(\widehat{s})].$$

The seller's profit function is $\Pi = PD(P)$, and this can be rewritten in terms of $\widehat{s}$ as $\Pi = p(\theta, \widehat{s})D(\widehat{s})$. The first-order condition is $\frac{\partial p(\theta,\widehat{s})}{\partial \widehat{s}} D(\widehat{s}) + p(\theta, \widehat{s})\frac{\partial D(\widehat{s})}{\partial \widehat{s}} \leq 0$. This gives us the optimal price $P^*(\theta)$. However, it is hard to find the sign of $\frac{\partial P^*(\theta)}{\partial \theta}$ in this general approach.

Let us look at the simplest case of this model in which a bad signal follows the uniform distribution on $[0, \underline{s}]$, and a good signal follows the uniform distribution on $[\overline{s}, 1]$, where $\overline{s} < \underline{s}$. When

$s > \underline{s}$, buyers can be sure that this is certainly good $H$, while when $s < \overline{s}$, this is certainly good $L$. When $s \in [\overline{s}, \underline{s}]$, the quality of the good is unclear. Since the likelihood ratio $\frac{1-\overline{s}}{\underline{s}}$ is constant, buyers purchase the good as long as $\frac{\underline{s}\theta}{\underline{s}\theta+(1-\overline{s})(1-\theta)} \geq P$.

**Proposition 7** *There exists a cutoff value $\widehat{\theta}$ so that*

$$P^* = \begin{cases} 1, & \text{if } \theta \leq \widehat{\theta}, \\ \frac{\underline{s}\theta}{\underline{s}\theta+(1-\overline{s})(1-\theta)}, & \text{if } \theta > \widehat{\theta}. \end{cases}$$

**Proof of Proposition 7**

*In the appendix.*

The result is very similar to our basic model. When the buyer has a relatively low valuation, the seller charges the maximum price 1 and serves only the buyer who can be sure of the high quality. By contrast, when the buyer's valuation is greater than the threshold $\widehat{\theta}$, the seller offers a lower price to serve the buyers who are uncertain of the quality. The reverse price discrimination, $P(\theta_L) > P(\theta_H)$, arises if $\theta_L \in [0, \widehat{\theta}]$ and $\theta_H \in (\widehat{\theta}, 1]$.

## 6 Concluding Remarks

This paper characterizes price discrimination under partially incomplete information in the sense that a buyer's prior valuation can be observed by the seller(s) but the buyer further draws a private signal which may give him or her additional information about a product sold by the seller(s). In this environment, we demonstrate the possibility that the buyer with a higher willingness to pay is offered a lower price and vice versa. We further show that this reverse price discrimination may arise even when there is competition between sellers. In the monopoly market, the seller only targets the buyers who draw a good signal by charging a high price when they are of a low type, whereas she wants to serve all of the buyers by offering a low price when they are of a high type. In the duopoly market, the sellers charge a higher price to the buyers with intermediate prior valuations than to the buyers with extreme prior valuations, since those buyers are likely to perceive that two competing products are more differentiated when they receive different signals from the sellers.

Our results from the basic model might be considered restrictive because the information quality should be intermediate, i.e., $\alpha \in [\alpha_1, \alpha_2]$ for the reverse price discrimination to arise. In other words, the signals buyers observe should be neither too precise nor too vague. However, we believe this is more realistic in many cases because a large number of goods are neither perfectly search goods nor perfectly experience goods. Consider a car sales market. Consumers are usually given some opportunities to inspect and test drive a car before the purchase is made, but it is still hard for them to completely figure out how well the car fits their taste. In this sense, the car sales market is a good example where reverse price discrimination may occur.

In our paper, the information structure $\alpha$ has been given exogenously. What happens if we relax this assumption so that the seller can choose the level of information to some extent? In particular, it would be interesting to ask whether the seller has incentives to engage in information discrimination. If we restrict our attention to the relevant case for reverse price discrimination, it can be easily understood that the seller has opposite incentives in providing information to different people. The seller would provide as much (precise) information as possible to the $L$ buyer, whereas she would provide as little (precise) information as possible to the $H$ buyer. Since the seller's optimal strategy for $L$ buyers is to charge a high price and serve only those who receive a good signal, she wants to allow them to update their beliefs more precisely so that they have a higher posterior valuation upon a good signal. On the contrary, for $H$ buyers, since the seller charges a low price and targets all buyers including the ones with a bad signal, the more precise information would only make the buyers with a bad signal more disappointed.[13] This result may be able to explain one interesting observation in Ayres and Siegelman (1995). They tested whether salespeople might discriminate between white and non-white buyers simply because of their animus or hostility. However, they found that salespeople actually spent more time with non-white consumers while offering higher prices, which is consistent with our argument. In future work, it would be interesting to analyze how price discrimination interacts with information discrimination in more detail.

# 7 Appendix

**Proof of Proposition 1**

$\pi_H(\theta_i, \alpha) \gtreqless \pi_L(\theta_i, \alpha) \Longrightarrow \alpha\theta_i \gtreqless \frac{(1-\alpha)\theta_i}{\alpha(1-\theta_i)+(1-\alpha)\theta_i} \Longrightarrow \theta_i\alpha^2(1-2\theta_i) + \alpha\left(\theta_i + \theta_i^2\right) - \theta_i \gtreqless 0$. Let $f(\alpha) = \theta_i\alpha^2(1-2\theta_i) + \alpha\left(\theta_i + \theta_i^2\right) - \theta_i$.

Case (1) $\theta = \theta_H > \frac{1}{2}$

Then, (a) $f(\alpha)$ is a concave function, (b) $f(\alpha)$ attains max value at $\alpha = \frac{\theta_H+1}{2(2\theta_H-1)} > 1$ and $f\left(\alpha = \frac{\theta_H+1}{2(2\theta_H-1)}\right) = \frac{(\theta_H-5)(\theta_H-1)\theta_H}{4(2\theta_H-1)} > 0$, (c) $f\left(\alpha = \frac{1}{2}\right) = \left(-\frac{1}{4}\right)\theta_H < 0$, (d) $f(\alpha = 1) = -\theta_H(\theta_H - 1) > 0$. So $\exists\alpha_2$ such that if $\alpha \in \left(\frac{1}{2}, \alpha_2\right]$, $f(\alpha) \leq 0$ and if $\alpha \in (\alpha_2, 1)$, $f(\alpha) > 0$. This implies that if $\alpha \in \left(\frac{1}{2}, \alpha_2\right]$, $\pi_H(\theta_H, \alpha) \leq \pi_L(\theta_H, \alpha)$ and if $\alpha \in (\alpha_2, 1)$, $\pi_H(\theta_H, \alpha) > \pi_L(\theta_H, \alpha)$.

[13] Technically, this argument is reminiscent of Johnson and Myatt (2006). They show that profits are "U-shaped" in the dispersion of demand. When the marginal consumer is above average, as in the case of the $L$ buyer, profits increase as the demand curve is more dispersed. In contrast, when the marginal consumer is below average, as in the case of the $H$ buyer, the less dispersion of the demand curve raises profits. In our model, the demand curve is dispersed by more precise information. A major difference in our model is that the degree of dispersion is systemically dependent on a buyer's prior valuation through Bayes' rule. Thus, we show that the seller provides the L buyer with more precise information to induce the dispersion of the demand curve, whereas she provides the H buyer with less precise information to reduce the dispersion of the demand curve.

Case (2) $\theta = \theta_L < \frac{1}{2}$

Then, (a) $f(\alpha)$ is a convex function, (b) $f(\alpha)$ attains min value at $\alpha = \frac{\theta_L+1}{2(2\theta_L-1)} < \frac{1}{2}$ and $f\left(\alpha = \frac{\theta_L+1}{2(2\theta_L-1)}\right) = \frac{(\theta_L-5)(\theta_L-1)\theta_L}{4(2\theta_L-1)} < 0$, (c) $f\left(\alpha = \frac{1}{2}\right) = \left(-\frac{1}{4}\right)\theta_L < 0$, (d) $f(\alpha = 1) = -\theta_L(\theta_L - 1) > 0$. So $\exists \alpha_1$ such that if $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $f(\alpha) < 0$ and if $\alpha \in [\alpha_1, 1)$, $f(\alpha) \geq 0$. This implies that if $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $\pi_H(\theta_L, \alpha) < \pi_L(\theta_L, \alpha)$ and if $\alpha \in [\alpha_1, 1)$, $\pi_H(\theta_L, \alpha) \geq \pi_L(\theta_L, \alpha)$.

Now let us define $\alpha^*(\theta_i) = \frac{1}{2\theta_i - 1}\left(\frac{1}{2}\theta_i - \frac{1}{2}\sqrt{-6\theta_i + \theta_i^2 + 5} + \frac{1}{2}\right)$. The computation yields that $\alpha_2 = \alpha^*(\theta_i = \theta_H)$ and $\alpha_1 = \alpha^*(\theta_i = \theta_L)$. Also

$$\frac{\partial(\alpha^*(\theta_i))}{\partial \theta_i} = -\frac{\left(5\theta_i - 7 + 3\sqrt{\theta_i^2 - 6\theta_i + 5}\right)}{2(2\theta_i - 1)^2\left(\sqrt{\theta_i^2 - 6\theta_i + 5}\right)} > 0$$

Here, $\left(3\sqrt{\theta^2 - 6\theta + 5}\right)^2 - (7 - 5\theta)^2 = (-4)(2\theta - 1)^2 < 0$, which implies $3\sqrt{\theta^2 - 6\theta + 5} < 7 - 5\theta$ because $3\sqrt{\theta^2 - 6\theta + 5} > 0$ and $7 - 5\theta > 0$ for $\theta \in [0, 1]$. As the numerator is negative, $\frac{\partial(\alpha^*(\theta_i))}{\partial \theta_i} > 0$. Then, this implies that $\alpha_2 > \alpha_1$ because $\theta_H > \frac{1}{2} > \theta_L$. Then, (i) If $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $\pi_H(\theta_i, \alpha) < \pi_L(\theta_i, \alpha)$. (ii) If $\alpha \in [\alpha_1, \alpha_2]$, $\pi_H(\theta_L, \alpha) > \pi_L(\theta_L, \alpha)$ and $\pi_H(\theta_H, \alpha) < \pi_L(\theta_H, \alpha)$. (iii) If $\alpha \in (\alpha_2, 1)$, $\pi_H(\theta_i, \alpha) > \pi_L(\theta_i, \alpha)$. Note that $\pi_H(\theta_i, \alpha)$ ($\pi_L(\theta_i, \alpha)$) is the profit when a seller charges $p_H(\theta_i, \alpha)$ ($p_L(\theta_i, \alpha)$) for type $i$. Then this proves Proposition 1. ■

**Proof of Proposition 3**

Let us define $\widetilde{\theta}_H(\theta_L)$ such that $p_L(\widetilde{\theta}_H(\theta_L), \alpha) = p_H(\theta_L, \alpha)$ for given $\theta_L$. Then, $p_L(\theta_H, \alpha) < p_H(\theta_L, \alpha)$ for $\theta_L < \theta_H < \widetilde{\theta}_H(\theta_L)$. By solving $\frac{\alpha\theta_L}{\alpha\theta_L + (1-\alpha)(1-\theta_L)} = \frac{(1-\alpha)\widetilde{\theta}_H(\theta_L)}{\alpha(1-\widetilde{\theta}_H(\theta_L)) + (1-\alpha)\widetilde{\theta}_H(\theta_L)}$ in terms of $\widetilde{\theta}_H(\theta_L)$, we obtain $\widetilde{\theta}_H(\theta_L) = \frac{\alpha^2\theta_L}{\alpha^2\theta_L + (1-\alpha)^2(1-\theta_L)}$. Or if we define $\widetilde{\theta}_L(\theta_H)$ such that $p_H(\widetilde{\theta}_L(\theta_H), \alpha) = p_L(\theta_H, \alpha)$ for given $\theta_H$, $p_L(\theta_H, \alpha) < p_H(\theta_L, \alpha)$ for $\widetilde{\theta}_L(\theta_H) < \theta_L < \theta_H$. By solving $\frac{(1-\alpha)\theta_H}{\alpha(1-\theta_H) + (1-\alpha)\theta_H} = \frac{\alpha\theta_L}{\alpha\theta_L + (1-\alpha)(1-\theta_L)}$, we obtain $\widetilde{\theta}_L(\theta_H) = \frac{(\alpha-1)^2\theta_H}{(\theta_H - 2\alpha\theta_H + \alpha^2)}$. ■

**Proof of Lemma 2**

(1) $x_{HL} = \frac{1}{2} + \frac{P^B - P^A + \Delta(\theta,\alpha)}{2t}$. (i) $x_{HL} > 0 \Longrightarrow P^B - P^A > -t - \Delta(\theta, \alpha)$, which is always true under our assumption. (ii) $x_{HL} < 1 \Longrightarrow P^B - P^A < t - \Delta(\theta, \alpha)$. $x_{LH} = \frac{1}{2} + \frac{P^B - P^A - \Delta(\theta,\alpha)}{2t}$. Then, (iii) $x_{LH} > 0 \Longrightarrow P^B - P^A > -t + \Delta(\theta, \alpha)$. (iv) $x_{LH} < 1 \Longrightarrow P^B - P^A < t + \Delta(\theta, \alpha)$, which is always true. Then, from (ii) and (iii), $-t + \Delta(\theta, \alpha) < P^B - P^A < t - \Delta(\theta, \alpha)$. So, $\left|P^B - P^A\right| < t - \Delta(\theta, \alpha) \Longrightarrow \Delta(\theta, \alpha) < t - \left|P^B - P^A\right|$. It also should be that $t > \Delta(\theta, \alpha)$ from $-t + \Delta(\theta, \alpha) < t - \Delta(\theta, \alpha)$. However, if $\Delta(\theta, \alpha) < t - \left|P^B - P^A\right|$, $\Delta(\theta, \alpha) < t$ is always true. Moreover, as we consider the symmetric equilibrium, both sellers' optimal prices should be same. Then, $\Delta(\theta, \alpha) < t - \left|P^B - P^A\right| \Longrightarrow \Delta(\theta, \alpha) < t$.

(2) From (1), $x_{HL} \geq 1 \Longrightarrow P^B - P^A \geq t - \Delta(\theta, \alpha)$ and $x_{LH} \leq 0 \Longrightarrow P^B - P^A \leq -t + \Delta(\theta, \alpha)$. So $t - \Delta(\theta, \alpha) \leq P^B - P^A \leq -t + \Delta(\theta, \alpha)$. Then, $\left|P^B - P^A\right| \leq -t + \Delta(\theta, \alpha) \Longrightarrow \Delta(\theta, \alpha) \geq$

$t+\left|P^B-P^A\right|$. It also should be that $t \leq \Delta(\theta,\alpha)$ from $t-\Delta(\theta,\alpha) \leq -t+\Delta(\theta,\alpha)$. However, if $\Delta(\theta,\alpha) \geq t+\left|P^B-P^A\right|$, $\Delta(\theta,\alpha) \geq t$ is always true. Also as we consider the symmetric equilibrium, $\Delta(\theta,\alpha) \geq t+\left|P^B-P^A\right| \Longrightarrow \Delta(\theta,\alpha) \geq t$. ■

**Proof of Lemma 3**

Case (1) $\Delta(\theta,\alpha) \leq t$

Seller $j$'s problem is

$$\underset{P_i}{Max}\ \pi^j = \left(\frac{t+P^{-j}-P^j}{2t}\right)P^j$$

The first-order condition is $\frac{\partial \pi^j}{\partial P^j} = \frac{1}{2t}\left(t-2P^j+P^{-j}\right)=0$. Solving the two first-order conditions, we obtain $P^{A*}=P^{B*}=t$.

Case (2) $\Delta(\theta,\alpha) > t$

Seller $j$'s problem is

$$\underset{P^j}{Max}\ \pi^j = \left(\left(\Pr(s_H)^2+\Pr(s_L)^2\right)\left(\frac{1}{2}+\frac{P^{-j}-P^j}{2t}\right)+\Pr(s_H)\Pr(s_L)\right)P^j$$

Then, the first-order condition is $\frac{\partial \pi^j}{\partial P^j} = \left(\Pr(s_H)^2+\Pr(s_L)^2\right)\frac{1}{2t}\left(t-2P^j+P^{-j}\right)+\Pr(s_H)\Pr(s_L) = 0$. Again, solving the two first-order conditions, we obtain $P^{A*}=P^{B*}=\left(\frac{(\Pr(s_H)+\Pr(s_L))^2}{(\Pr(s_H)^2+\Pr(s_L)^2)}\right)t$. ■

**Proof of Proposition 4**

Let us check the condition $\Delta(\theta,\alpha) \gtreqless t$. Here, $\Delta(\theta,\alpha) \equiv p_H(\theta,\alpha)-p_L(\theta,\alpha) = \frac{(\theta-1)(2\alpha-1)\theta}{(2\alpha\theta-\theta-\alpha+1)(2\alpha\theta-\theta-\alpha)}$. We obtain $\frac{\partial\Delta(\theta,\alpha)}{\partial\theta} = \frac{(2\theta-1)(\alpha-1)(2\alpha-1)\alpha}{(2\theta\alpha-\alpha-\theta+1)^2(2\theta\alpha-\alpha-\theta)^2}$.

Case (1) $\theta > \frac{1}{2}$

(i) $\frac{\partial\Delta(\theta,\alpha)}{\partial\theta} < 0$, so $\Delta(\theta,\alpha)$ is a decreasing function, (ii) $\Delta\left(\theta=\frac{1}{2},\alpha\right) = (2\alpha-1) > 0$, and (iii) $\Delta(\theta=1,\alpha)=0$. So if $(2\alpha-1) > t$, $\exists\bar{\theta}$ such that if $\theta \in (\frac{1}{2},\bar{\theta}]$, $\Delta(\theta,\alpha) > t$ and if $\theta \in [\bar{\theta},1)$, $\Delta(\theta,\alpha) < t$. On the other hand, if $(2\alpha-1) < t$, $\Delta(\theta,\alpha) < t$ for all $\theta \in \left(\frac{1}{2},1\right)$.

Case (2) $\theta < \frac{1}{2}$

(i) $\frac{\partial\Delta(\theta,\alpha)}{\partial\theta} > 0$, so $\Delta(\theta,\alpha)$ is an increasing function, (ii) $\Delta(\theta=0)=0$, (iii) $\Delta\left(\theta=\frac{1}{2},\alpha\right) = (2\alpha-1)$. So if $(2\alpha-1) > t$, $\exists\underline{\theta}$ s.t. if $\theta \in (0,\underline{\theta}]$, $\Delta(\theta,\alpha) < t$ and if $\theta \in [\underline{\theta},\frac{1}{2})$, $\Delta(\theta,\alpha) > t$. On the other hand, if $(2\alpha-1) < t$, $\Delta(\theta,\alpha) < t$ for all $\theta \in \left(0,\frac{1}{2}\right)$.

In both cases, the condition that $(2\alpha-1) \gtreqless t \Longrightarrow \alpha \gtreqless \frac{t+1}{2}$ matters. Note that $\frac{t+1}{2}-\frac{1}{2}=\frac{1}{2}t>0$ and $\frac{t+1}{2}-1=\frac{1}{2}(t-1)$. Then we have the following result. *(1) if $t>1$, always for all $\theta \in (0,1)$, $\Delta(\theta,\alpha) < t$. (2) if $t<1$, $\exists\alpha^* = \frac{t+1}{2}$ such that if $\alpha \in \left(\frac{1}{2},\frac{t+1}{2}\right)$, for all $\theta \in (0,1)$, $\Delta(\theta,\alpha) < t$. On the other hand, if $\alpha \in \left(\frac{t+1}{2},1\right)$, there exist $\underline{\theta}$ and $\bar{\theta}$ such that if $\theta \in (0,\underline{\theta})$ or $\theta \in (\bar{\theta},1)$, $\Delta(\theta,\alpha) < t$ and if $\theta \in (\underline{\theta},\bar{\theta})$, $\Delta(\theta,\alpha) > t$.* We already know that if $\Delta(\theta,\alpha) < t$, the optimal price is $P^{A*}=P^{B*}=t$ and if $\Delta(\theta,\alpha) \geq t$, the optimal price is $P^{A*}=P^{B*}=t\left(\frac{(\Pr(s_H)+\Pr(s_L))^2}{(\Pr(s_H)^2+\Pr(s_L)^2)}\right)$. ■

**Proof of Lemma 4**

Case (1) $P = p_H(\theta) - \frac{D}{\Pr(s_H)}$

In this case, the optimal price is $P^* = \frac{p_H(\theta,\alpha)}{2} = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and it is binding only if $p_L(\theta,\alpha) < P^* < p_H(\theta,\alpha)$ is satisfied. It is always true that $P^* < p_H(\theta,\alpha)$ because $P^* - p_H(\theta,\alpha) = -\frac{\alpha\theta}{2(\theta(2\alpha-1)+(1-\alpha))} < 0$. Also $P^* - p_L(\theta,\alpha) = -\frac{\left(2\theta+4\alpha-5\theta\alpha-\alpha^2+2\theta\alpha^2-2\right)\theta}{2(\theta(2\alpha-1)+(1-\alpha))(2\theta\alpha-\alpha-\theta)}$ where $2\theta\alpha-\alpha-\theta<0$ for all $\alpha \in \left(\frac{1}{2},1\right)$ and $\theta \in (0,1)$. The analysis of the numerator yields the following result.

Result 1. $p_L(\theta,\alpha) < P^*$ holds if $\alpha \in \left(2-\sqrt{2},1\right)$ and $\theta \in \left(0,\frac{\alpha^2-4\alpha+2}{2\alpha^2-5\alpha+2}\right)$.

Case (2) $P = \theta - D$

In this case, the optimal price is $P^* = \frac{\theta}{2}$ and it is binding only if $0 < P^* < p_L(\theta,\alpha) = \frac{(1-\alpha)\theta}{\alpha(1-\theta)+(1-\alpha)\theta}$ is satisfied. Here, $P^* - p_L(\theta,\alpha) = \frac{(2\theta\alpha-3\alpha-\theta+2)\theta}{2(2\theta\alpha-\alpha-\theta)}$ where $2\theta\alpha-\alpha-\theta<0$ always. Then the analysis of the numerator yields the following result.

Result 2. $P^* < p_L(\theta,\alpha)$ holds in following two cases: (i) If $\alpha \in \left(\frac{2}{3},1\right)$ and $\theta \in \left(\frac{3\alpha-2}{2\alpha-1},1\right)$ or (ii) if $\alpha \in \left(\frac{1}{2},\frac{2}{3}\right)$ and $\theta \in \left(\frac{1}{2},1\right)$.

Then, from Results 1 and 2, the optimal price $P^*$ can be described as follows.

Result 3. (1) If $\alpha \in \left(\frac{1}{2},2-\sqrt{2}\right)$, $P^* = \frac{\theta}{2}$. (2) Suppose that $\alpha \in \left(2-\sqrt{2},\frac{2}{3}\right)$. (i) If $\theta \in \left(0,\frac{\alpha^2-4\alpha+2}{2\alpha^2-5\alpha+2}\right)$, there exist two candidates $P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and $\frac{\theta}{2}$. (ii) If $\theta \in \left(\frac{\alpha^2-4\alpha+2}{2\alpha^2-5\alpha+2},1\right)$, $P^* = \frac{\theta}{2}$. (3) Suppose that $\alpha \in \left(\frac{2}{3},1\right)$. (i) If $\theta \in \left(0,\frac{3\alpha-2}{2\alpha-1}\right)$, $P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$, (ii) if $\theta \in \left(\frac{3\alpha-2}{2\alpha-1},\frac{\alpha^2-4\alpha+2}{2\alpha^2-5\alpha+2}\right)$, there exist two candidates: $P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and $\frac{\theta}{2}$, (iii) if $\theta \in \left(\frac{\alpha^2-4\alpha+2}{2\alpha^2-5\alpha+2},1\right)$, $P^* = \frac{\theta}{2}$.

Now, for given two candidates $P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and $\frac{\theta}{2}$, we check which price yields a greater profit to the seller. When $P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$, the profit is denoted by $\pi(P^* = \frac{p_H(\theta)}{2}) = \frac{\alpha^2\theta^2}{4(2\theta\alpha-\alpha-\theta+1)}$ and when $P^* = \frac{\theta}{2}$, the profit is denoted by $\pi(P^* = \frac{\theta}{2}) = \frac{\theta^2}{4}$. The comparison of both profits yields the following result.

Result 4. (1) If $\alpha < \frac{\sqrt{5}-1}{2}$, $\pi(P^* = \frac{p_H(\theta,\alpha)}{2}) < \pi(P^* = \frac{\theta}{2}) \Longrightarrow P^* = \frac{\theta}{2}$. (2) When $\alpha > \frac{\sqrt{5}-1}{2}$, if $\theta \in \left(0,\frac{\alpha+\alpha^2-1}{2\alpha-1}\right)$, $\pi(P^* = \frac{p_H(\theta,\alpha)}{2}) > \pi(P^* = \frac{\theta}{2}) \Longrightarrow P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and if $\theta \in \left(\frac{\alpha+\alpha^2-1}{2\alpha-1},1\right)$, $\pi(P^* = \frac{p_H(\theta,\alpha)}{2}) < \pi(P^* = \frac{\theta}{2}) \Longrightarrow P^* = \frac{\theta}{2}$.

We apply Result 4 to 2-(i) and 3-(ii), Result 3 can be simplified as Lemma 9.

(1) Suppose that $\alpha \in \left(\frac{1}{2},\frac{\sqrt{5}-1}{2}\right)$. Then $P^* = \frac{\theta}{2}$.

(2) Suppose that $\alpha \in \left(\frac{\sqrt{5}-1}{2},1\right)$. If $\theta \in \left(0,\frac{\alpha+\alpha^2-1}{2\alpha-1}\right)$, $P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and if $\theta \in \left(\frac{\alpha+\alpha^2-1}{2\alpha-1},1\right)$, $P^* = \frac{\theta}{2}$. ■

**Proof of Proposition 7**

The seller's pricing decision is either 1 or $\frac{\underline{s}\theta}{\underline{s}\theta+(1-\overline{s})(1-\theta)}$. When $P=1$, the fraction of buyers $\frac{1-\underline{s}}{1-\overline{s}}\theta$ purchase the good. This is the probability that buyers draw a signal from $(\underline{s},1]$. On the other hand, when $P=\frac{\underline{s}\theta}{\underline{s}\theta+(1-\overline{s})(1-\theta)}$, the fraction of buyers $\theta+(1-\theta)\frac{\underline{s}-\overline{s}}{\underline{s}}$ purchase the good. This is the probability that buyers draw a signal from $[\overline{s},1]$. We can easily compute the seller's profit at each price as follows.

$$\begin{aligned}\pi_H &= \frac{1-\underline{s}}{1-\overline{s}}\theta \qquad\qquad \text{at } P=1, \text{ and}\\ \pi_L &= \frac{\underline{s}\theta+(\underline{s}-\overline{s})(1-\theta)}{\underline{s}\theta+(1-\overline{s})(1-\theta)}\theta \text{ at } P=\frac{\underline{s}\theta}{\underline{s}\theta+(1-\overline{s})(1-\theta)}.\end{aligned}$$

It can be shown that $\pi_H/\pi_L$ is monotonically decreasing in $\theta$, with $\pi_H/\pi_L=\frac{1-\underline{s}}{\underline{s}-\overline{s}}$ at $\theta=0$ and $\pi_H/\pi_L=\frac{1-\underline{s}}{1-\overline{s}}$ at $\theta=1$. Note $\frac{1-\underline{s}}{\underline{s}-\overline{s}}>1>\frac{1-\underline{s}}{1-\overline{s}}$ under our assumption $\overline{s}<\underline{s}$. Thus, there should be a unique $\widehat{\theta}$ such that $\pi_H \gtreqless \pi_L$ as $\theta \lesseqgtr \widehat{\theta}$, where $\pi_H(\widehat{\theta})=\pi_L(\widehat{\theta})$. ■

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